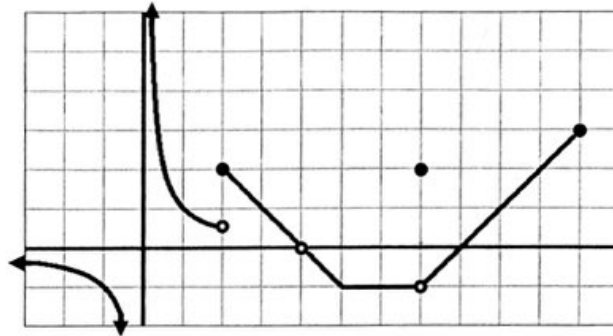


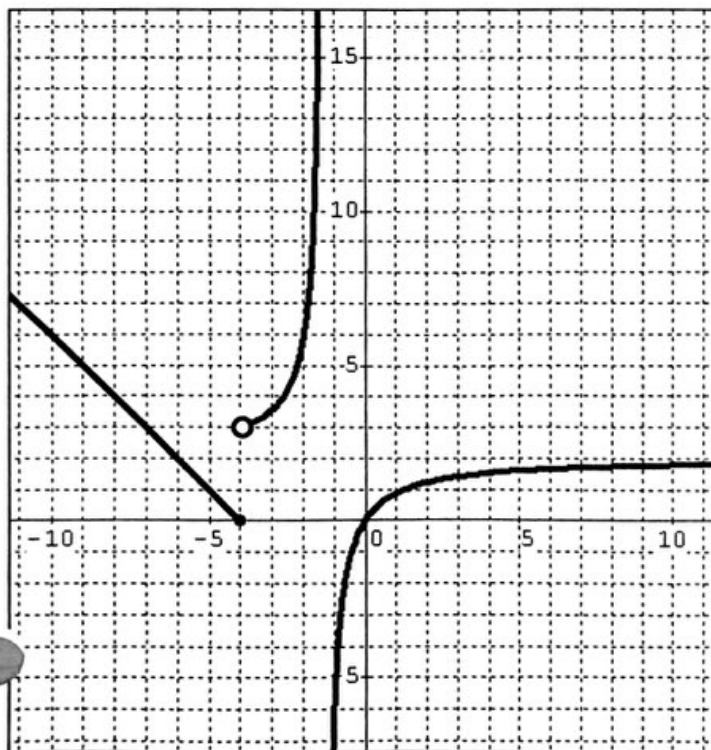
Limits Graphically

For questions #1- 18, refer to the graph of $f(x)$ below to evaluate the following limits.



- | | | |
|---|--|---|
| 1) <u>∞</u> $\lim_{x \rightarrow 0^+} f(x)$ | 2) <u>$-\infty$</u> $\lim_{x \rightarrow 0^-} f(x)$ | 3) <u>DNE</u> $\lim_{x \rightarrow 0} f(x)$ |
| 4) <u>2</u> $\lim_{x \rightarrow 2^+} f(x)$ | 5) <u>$\frac{1}{2}$</u> $\lim_{x \rightarrow 2^-} f(x)$ | 6) <u>DNE</u> $\lim_{x \rightarrow 2} f(x)$ |
| 7) <u>-1</u> $\lim_{x \rightarrow 7^+} f(x)$ | 8) <u>-1</u> $\lim_{x \rightarrow 7^-} f(x)$ | 9) <u>-1</u> $\lim_{x \rightarrow 7} f(x)$ |
| 10) <u>DNE</u> $\lim_{x \rightarrow 11^+} f(x)$ | 11) <u>3</u> $\lim_{x \rightarrow 11^-} f(x)$ | 12) <u>DNE</u> $\lim_{x \rightarrow 11} f(x)$ |
| 13) <u>0</u> $\lim_{x \rightarrow 4} f(x)$ | 14) <u>-1</u> $\lim_{x \rightarrow 5} f(x)$ | 15) <u>und</u> $f(0)$ |
| 16) <u>2</u> $f(2)$ | 17) <u>und</u> $f(4)$ | 18) <u>2</u> $f(7)$ |

For questions # 19 – 30, refer to the graph of $f(x)$ below to evaluate the following limits.



- | | |
|---|--|
| 19) <u>DNE</u> $\lim_{x \rightarrow 4} f(x)$ | 20) <u>DNE</u> $\lim_{x \rightarrow -1} f(x)$ |
| 21) <u>0</u> $\lim_{x \rightarrow 0} f(x)$ | 22) <u>∞</u> $\lim_{x \rightarrow -\infty} f(x)$ |
| 23) <u>2</u> $\lim_{x \rightarrow \infty} f(x)$ | 24) <u>$-\infty$</u> $\lim_{x \rightarrow -1^+} f(x)$ |
| 25) <u>∞</u> $\lim_{x \rightarrow -1^-} f(x)$ | 26) <u>3</u> $\lim_{x \rightarrow -4^+} f(x)$ |
| 27) <u>0</u> $\lim_{x \rightarrow -4^-} f(x)$ | 28) <u>0</u> $f(-4)$ |
| 29) <u>0</u> $f(0)$ | 30) <u>und</u> $f(-1)$ |

LIMITS ALGEBRAICALLY

Find each limit algebraically. Complete all work on a separate sheet of paper. Place final answers on the line provided.

12 1) $\lim_{x \rightarrow 1} (12x^3 + x^2 - 1)$

$$12(1)^3 + (1)^2 - 1$$

$$12$$

1/4 2) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3}$

$$\frac{\sqrt{3+1} - 2}{3-3} = \frac{0}{0}$$

$$\frac{(\sqrt{x+1} - 2)(\sqrt{x+1} + 2)}{(x-3)(\sqrt{x+1} + 2)}$$

$$\frac{x+1-4}{(x-3)(\sqrt{x+1} + 2)} = \frac{x-3}{(x-3)(\sqrt{x+1} + 2)}$$

$$\frac{1}{\sqrt{x+1} + 2} = \frac{1}{\sqrt{3+1} + 2} = \frac{1}{4}$$

1/5 3) $\lim_{x \rightarrow \infty} \frac{(x) \frac{1}{x} - \frac{3}{x^2}}{5(x^2) \frac{6}{x^4}} = \frac{x-3}{5x^3+6}$

$$\frac{x-3}{x^2} \cdot \frac{x^2}{5x^3+6}$$

$$\frac{x^3-3x^2}{5x^3+6}$$

Num = Den

-∞ 4) $\lim_{x \rightarrow -\infty} (2 - x^2)$

$$\lim_{x \rightarrow -\infty} -x^2 + 2$$

$$\begin{array}{c} \curvearrowright \quad \curvearrowright \\ -\infty \end{array}$$

0 5) $\lim_{x \rightarrow \infty} \frac{2x+7}{x^2-x}$

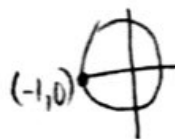
Num < Den

0 6) $\lim_{x \rightarrow \pi} (\cos x \sin x)$

$$\cos(\pi) \sin(\pi)$$

$$-1 \cdot 0$$

$$0$$



0 7) $\lim_{x \rightarrow \infty} \frac{x^{-4} - 2x^{-5}}{x^{-2} + 4x^{-6}}$

$$\frac{(x) \frac{1}{x^4} - \frac{2}{x^5}}{(x) \frac{1}{x^2} + \frac{4}{x^6}}$$

$$\frac{x-2}{x^5} = \frac{x-2}{x^5} \cdot \frac{x^6}{x^6} = \frac{x^2-2x}{x^4+4}$$

$$\frac{x^2-2x}{x^4+4}$$

Num < Den

1/8 8) $\lim_{x \rightarrow 16} \frac{\sqrt{x}-4}{x-16}$

$$\frac{\sqrt{16}-4}{16-16} = \frac{0}{0}$$

$$\frac{(\sqrt{x}-4)(\sqrt{x}+4)}{(x-16)(\sqrt{x}+4)}$$

$$\frac{x-16}{(x-16)(\sqrt{x}+4)}$$

$$\frac{1}{\sqrt{x}+4}$$

$$\frac{1}{\sqrt{16}+4} = \frac{1}{8}$$

15 9) $\lim_{x \rightarrow 5} \frac{2x^2 - 5x - 25}{x-5}$

$$\frac{2(5)^2 - 5(5) - 25}{5-5} = \frac{0}{0}$$

$$\frac{(2x+5)(x-5)}{(x-5)}$$

$$2x+5$$

$$2(5)+5$$

$$15$$

$$\frac{1}{2} \quad 10) \lim_{x \rightarrow \infty} \frac{3x^3 - x + 1}{6x^3 + 2x^2 - 7}$$

Num = Den

$$\frac{3}{6}$$

$$\frac{1}{2}$$

$$+\infty \quad 11) \lim_{x \rightarrow -\infty} \frac{3 - 4x - x^2}{x + 1}$$

$$\frac{-x^2 - 4x + 3}{x + 1}$$

Num > Den ($\pm \infty$)

$$\frac{-(-100)^2 - 4(-100) + 3}{(-100) + 1}$$

$$\frac{-}{-} = \frac{\infty}{\infty}$$

$$-3 \quad 12) \lim_{x \rightarrow 0} \frac{6x - 9}{x^3 - 12x + 3}$$

$$\frac{6(0) - 9}{0^3 - 12(0) + 3}$$

$$\frac{-9}{3}$$

$$-3$$

$$\frac{1}{4} \quad 13) \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$$

(hole) $\frac{\sqrt{4} - 2}{4 - 4} = \frac{0}{0}$

$$\frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{x - 4 (\sqrt{x} + 2)}$$

$$\frac{\cancel{x-4}}{(\cancel{x-4})(\sqrt{x} + 2)}$$

$$\frac{1}{\sqrt{x} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$$

$$1 \quad 14) \lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x - 3}$$

$$\frac{(3)^2 - 5(3) + 6}{3 - 3} = \frac{0}{0}$$

$$\frac{(x-2)(x-3)}{(x-3)}$$

$$x - 2$$

$$3 - 2$$

$$1$$

$$\frac{6}{7} \quad 15) \lim_{x \rightarrow 5} \frac{x+1}{x+2}$$

$$\frac{5+1}{5+2}$$

$$\frac{6}{7}$$

$$1 \quad 16) \lim_{x \rightarrow \infty} \frac{x^2 - 4x + 4}{x^2 + x - 6}$$

Num = Den

1

$$-\infty \quad 17) \lim_{x \rightarrow \infty} (3 - x^3) =$$

$$\lim_{x \rightarrow \infty} (-x^3 + 3)$$

↑

↓

$-\infty$

$$\infty \quad 18) \lim_{x \rightarrow 4} \frac{5x - 1}{(x - 4)^2} =$$

$$\frac{5(4) - 1}{(4 - 4)^2}$$

$$\frac{19}{0} \quad (\text{VA})$$

$$\lim_{x \rightarrow 4^+} = \frac{5(4.1) - 1}{(4.1 - 4)^2} = \frac{+}{+} = \infty$$

$$\lim_{x \rightarrow 4^-} = \frac{5(3.9) - 1}{(3.9 - 4)^2} = \frac{+}{+} = \infty$$

$$\lim_{x \rightarrow 4} = \lim_{x \rightarrow 4^+} = \lim_{x \rightarrow 4^-} = \infty$$

$$\infty \quad 19) \lim_{x \rightarrow \infty} \frac{4x^2 - 2x + 3}{3x - 1} =$$

NUM > Den
NO H.A. ($\pm \infty$)

$$\frac{4(100)^2 - 2(100) + 3}{3(100)}$$

$$\frac{+}{+} = \infty$$

$$\frac{1}{6} \quad 20) \lim_{x \rightarrow 8} \frac{\sqrt{x+1} - 3}{x - 8} =$$

$$\frac{\sqrt{8+1} - 3}{8 - 8} = \frac{0}{0}$$

$$\frac{(\sqrt{x+1} - 3)(\sqrt{x+1} + 3)}{(x - 8)}$$

$$\frac{x+1-9}{(x-8)(\sqrt{x+1}+3)} = \frac{x-8}{(x-8)(\sqrt{x+1}+3)}$$

$$\frac{1}{\sqrt{x+1}+3}$$

$$\frac{1}{\sqrt{8+1}+3} = \frac{1}{6}$$

$$-5 \quad 23) \lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x + 1} =$$

$$\frac{2(-1) - (-1)^2 - 3}{-1 + 1} = \frac{0}{0}$$

$$\frac{(2x-3)(x+1)}{x+1}$$

$$2x - 3$$

$$2(-1) - 3$$

$$\frac{-2 - 3}{-1} = -5$$

$$-2 \quad 26) \lim_{x \rightarrow \infty} \frac{\frac{8}{x^2} - \frac{2}{x^2}}{\frac{7}{x^3} - \frac{4}{x^3}}$$

$$\frac{8x^2 - 2}{x^2}$$

$$\frac{7 - 4x^3}{x^3}$$

$$\frac{(8x^2 - 2) \cdot x^3}{x^2 \cdot (7 - 4x^3)}$$

$$\frac{8x^3 - 2x}{-4x^3 + 7}$$

$$\frac{8}{-4} = -2$$

$$-2 \quad 21) \lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} =$$

$$\frac{(-1)^2 - 1}{-1 + 1} = \frac{0}{0}$$

$$\frac{(x+1)(x-1)}{(x+1)}$$

$$\frac{x-1}{-1-1} = -2$$

$$\infty \quad 22) \lim_{x \rightarrow \infty} \frac{\frac{1}{x^2}}{-\frac{1}{x^4} + \frac{1}{x^3}}$$

$$\frac{\frac{1}{x^2}}{\frac{x-1}{x^4}} = \frac{1}{x^2} \cdot \frac{x^4}{x-1} = \frac{x^2}{x-1}$$

$$\lim_{x \rightarrow \infty} \frac{x^2}{-x-1} \quad \text{Num > Den, NO H.A. ($\pm \infty$)}$$

$$\frac{(100)^2}{(100) - 1} = \frac{+}{+}$$

$$\infty$$

$$-\frac{1}{14} \quad 25) \lim_{x \rightarrow -7} \frac{x+7}{x^2 - 49} =$$

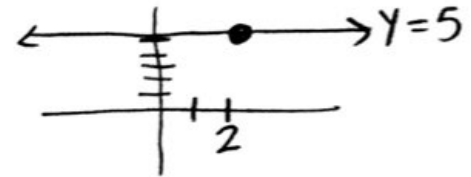
$$\frac{(-7)+7}{(-7)^2 - 49} = \frac{0}{0}$$

$$\frac{(x+7)}{(x+7)(x-7)}$$

$$\frac{1}{x-7}$$

$$\frac{1}{-7-7} = -\frac{1}{14}$$

$$5 \quad 24) \lim_{x \rightarrow 2} 5 =$$



$$0 \quad 27) \lim_{x \rightarrow \frac{\pi}{2}} (\sin^2 x - 1) =$$

$$\left[\sin\left(\frac{\pi}{2}\right) \right]^2 - 1$$



$$(1)^2 - 1$$

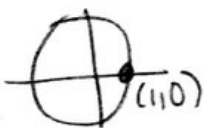
$$1 - 1$$

$$0$$

$$\frac{3}{2} \quad 28) \lim_{x \rightarrow 0} \frac{3 - \sin x}{\cos x + 1} =$$

$$\frac{3 - \sin(0)}{\cos(0) + 1}$$

$$\frac{3 - 0}{1 + 1} = \frac{3}{2}$$



$$4 \quad 29) \lim_{x \rightarrow 4} \frac{4 - x}{2 - \sqrt{x}}$$

$$\frac{4 - 4}{2 - \sqrt{4}} = \frac{0}{0}$$

$$\frac{(4-x)(2+\sqrt{x})}{(2-\sqrt{x})(2+\sqrt{x})}$$

$$\frac{4-x}{2+\sqrt{x}}$$

$$-16 \quad 32) \lim_{x \rightarrow 0} \frac{(x-8)^2 - 64}{x}$$

$$\frac{(0-8)^2 - 64}{0} = \frac{0}{0}$$

$$\frac{(x-8)(x-8) - 64}{x^2 - 16x + 64 - 64}$$

$$\frac{x(x-16)}{x}$$

$$\frac{x-16}{0-16} = -16$$

$$9 \quad 30) \lim_{x \rightarrow 9} x$$

$$x$$

$$9$$

$$-\frac{1}{3} \quad 31) \lim_{x \rightarrow 0} \frac{x}{x^2 - 3x}$$

$$\frac{0}{0^2 - 3(0)} = \frac{0}{0}$$

$$\frac{x}{x(x-3)}$$

$$\frac{1}{x-3}$$

$$\frac{1}{0-3} = -\frac{1}{3}$$

$$-12 \quad 34) \lim_{x \rightarrow 0} \frac{(x-6)^2 - 36}{x}$$

$$\frac{(0-6)^2 - 36}{0} = \frac{0}{0}$$

$$\frac{(x-6)(x-6) - 36}{x^2 - 12x + 36 - 36}$$

$$\frac{x(x-12)}{x}$$

$$\frac{x-12}{-12}$$

$$0 \quad 35) \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x - 1}$$

$$\frac{(1)^2 - 2(1) + 1}{1 - 1} = \frac{0}{0}$$

$$\frac{(x-1)(x-1)}{(x-1)}$$

$$(x-1)$$

$$1 - 1$$

$$0$$

$$\frac{5}{6} \quad 33) \lim_{x \rightarrow -3} \frac{2x+1}{x-3}$$

$$\frac{2(-3)+1}{-3-3}$$

$$-\frac{5}{6}$$

$$\frac{5}{6}$$

$$\star \text{ SKIP} \quad 36) \lim_{x \rightarrow -\pi} \frac{\sin(x+\pi)}{x+\pi} =$$

$$\frac{\sin(-\pi+\pi)}{-\pi+\pi}$$

$$\frac{\sin(0)}{0} = \frac{0}{0}$$

$$\underline{-\infty} \quad 37) \lim_{x \rightarrow \infty} \frac{(-3x-1)(-2x+4)(-5x-3)}{(-6x-1)(-2x+3)}$$

$$\frac{x^3}{x^2} \quad \text{Num} > \text{Den} \quad (\pm \infty)$$

$$\frac{[-3(100)-1][-2(100)+4][-5(100)-3]}{[-6(100)-1][-2(100)+3]}$$

$$\frac{(-)(-)(-)}{(-)(-)} = \frac{-}{+} = -\infty$$

$$\frac{(-)(-)(-)}{(-)(-)} = \frac{-}{+} = -\infty$$

$$\underline{-\infty} \quad 38) \lim_{x \rightarrow -\infty} \frac{-3x^3 - 6x^2 + 4x - 3}{-2x^2 + 5x}$$

$$\text{Num} > \text{Den} \quad (\pm \infty)$$

$$\frac{-3(-100)^3 - 6(-100)^2 + 4(-100) - 3}{-2(-100)^2 + 5(-100)}$$

$$\frac{+}{-}$$

$$-\infty$$

$$\underline{\text{DNE}} \quad 39) \lim_{x \rightarrow 4} f(x), \quad f(x) = \begin{cases} \frac{1}{2}x - 1, & x \geq 4 \\ 2x - 1, & x < 4 \end{cases}$$

$$\lim_{x \rightarrow 4^-} 2(4) - 1 = 7$$

$$\lim_{x \rightarrow 4^+} \frac{1}{2}(4) - 1 = 1$$

$$\lim_{x \rightarrow 4^-} \neq \lim_{x \rightarrow 4^+}$$

DNE

$$\underline{0} \quad 40) \lim_{x \rightarrow -2} f(x), \quad f(x) = \begin{cases} -x^2 + 4, & x > -2 \\ 3x + 6, & x < -2 \end{cases}$$

$$\lim_{x \rightarrow -2^+} -(-2)^2 + 4 = 0$$

$$\lim_{x \rightarrow -2^-} 3(-2) + 6 = 0$$

$$\lim_{x \rightarrow -2^-} \doteq \lim_{x \rightarrow -2^+}$$

$$0 = 0$$

$$0$$