Lesson 1.11 Objective: SWBAT find inverse functions and determine they are inverses.

Kickoff

1) If $g(x) = x^2-3x+1$ and $f(x) = 2x-1$ find $g(f(x))$.

$(2x-1)^2 - 3(2x-1) + 1$

$(2x-1)(2x-1) - 6x + 3 + 1$

$4x^2 - 2x - 2x + 1 - 6x + 3 + 1$

$4x^2 - 10x + 5$

---

1) If $f(x) = 5x^2 - 1$ and $g(x) = 3x - 1$, find $g(f(1))$.

$f(1) = 5(1)^2 - 1 \quad g(4) = 3(4) - 1$

$f(1) = 4 \quad g(4) = 11$

$\boxed{g(f(1)) = 11}$

2) If $f(x) = 2x + 4$ and $g(x) = x^2 + 1$, find $(f \circ g)(3)$.

$f(10) = 2(10) + 4 \quad g(3) = (3)^2 + 1$

$f(10) = 24 \quad g(3) = 10$

$\boxed{(f \circ g)(3) = 24}$

3) If $f(x) = 2x - 5$ and $g(x) = \sqrt{x}$, evaluate $(f \circ g)(36)$.

$f(6) = 2(6) - 5 \quad g(36) = \sqrt{36}$

$f(6) = 7 \quad g(36) = 6$

$\boxed{(f \circ g)(36) = 7}$

4) If $f(x) = \frac{2}{\sqrt{5 - x^2}}$ and $g(x) = x + 1$, evaluate $(f \circ g)(0)$.

$g(0) = 0 + 1$

$g(0) = 1$

$f(1) = \frac{2}{\sqrt{5-(1)^2}}$

$f(1) = \frac{2}{\sqrt{4}}$

$f(1) = \frac{2}{2} = 1$

$\boxed{(f \circ g)(0) = 1}$

---

5) If $f(x) = x^2 + 4$ and $g(x) = 2x + 3$, find $f(g(-2))$.

$f(-1) = (-1)^2 + 4 \quad g(-2) = 2(-2) + 3$

$f(-1) = 5 \quad g(-2) = -1$

$\boxed{f(g(-2)) = 5}$

6) If $f(x) = 5x - 2$ and $g(x) = \sqrt[3]{x}$, find $(f \circ g)(-8)$.

$f(-2) = 5(-2) - 2 \quad g(-8) = \sqrt[3]{-8}$

$f(-2) = -12 \quad g(-8) = -2$

$\boxed{(f \circ g)(-8) = -12}$

7) If $f(x) = 2^x - 1$ and $g(x) = x^2 - 1$, find $(f \circ g)(3)$.

$f(8) = 2^8 - 1 \quad g(3) = (3)^2 - 1$

$f(8) = 255 \quad g(3) = 8$

$\boxed{(f \circ g)(3) = 255}$

8) If $f(x) = x - 2$ and $g(x) = x^2$, find $f(g(3))$.

$f(9) = 9 - 2 \quad g(3) = (3)^2$

$f(9) = 7 \quad g(3) = 9$

$\boxed{f(g(3)) = 7}$

---

9) If $f(x) = 3x - 5$ and $g(x) = x - 9$, find $(f \circ g)(x)$.

$f(x-9) = 3(x-9) - 5$

$3x - 27 - 5$

$3x - 32$

$\boxed{(f \circ g)(x) = 3x - 32}$

10) If $f(x) = x^2 - 5$ and $g(x) = 6x$, find $g(f(x))$

$g(x^2-5) = 6(x^2-5)$

$6x^2 - 30$

$\boxed{g(f(x)) = 6x^2 - 30}$

11) If $f(x) = 3x + 5$ and $g(x) = x^2 + 1$, find $g(f(x))$.

$g(3x+5) = (3x+5)^2 + 1$

$(3x+5)(3x+5) + 1$

$9x^2 + 30x + 25 + 1$

$\boxed{g(f(x)) = 9x^2 + 30x + 26}$

12) If $f(x) = \frac{2}{x+3}$ and $g(x) = \frac{1}{x}$, then $(g \circ f)(x)$

$g\left(\frac{2}{x+3}\right) = \frac{1}{\frac{2}{x+3}}$

$1 \rightarrow \frac{x+3}{2} = \frac{x+3}{2}$

$\boxed{(g \circ f)(x) = \frac{x+3}{2}}$

---

13) If $f(x) = 2x - 1$ and $g(x) = 3x + 5$, find $(f \circ g)(x)$

$f(3x+5) = 2(3x+5) - 1$

$6x + 10 - 1$

$6x + 9$

$\boxed{(f \circ g)(x) = 6x + 9}$

14) If $f(x) = x^2$ and $g(x) = 2x + 1$ find $(f \circ g)(x)$?

$f(2x+1) = (2x+1)^2 \quad f(g(x))$

$(2x+1)(2x+1)$

$4x^2 + 4x + 1$

$\boxed{(f \circ g)(x) = 4x^2 + 4x + 1}$

---

15) Given: $f(x) = \sqrt{2x + 5}$ and $g(x) = 6x - 3$,

a. Find $g(f(10))$

b. Find $(f \circ g)(x)$.

a) $f(10) = \sqrt{2(10) + 5} \quad g(5) = 6(5) - 3$

$f(10) = \sqrt{25} \quad g(5) = 30 - 3$

$f(10) = 5 \quad g(5) = 27$

b) $f(6x-3) = \sqrt{2(6x-3)}$

$f(6x-3) = \sqrt{12x - 6 + 5}$

$\sqrt{12x - 1}$

16) If $f(x) = x^{\frac{2}{3}}$ and $g(x) = 8x^{-\frac{1}{2}}$,

a. Find $(f \circ g)(x)$

Find $(f \circ g)(27)$.

$g(27) = 8(27)^{-\frac{1}{2}}$

$f\left(8(27)^{-\frac{1}{2}}\right) = \left(8(27)^{-\frac{1}{2}}\right)^{\frac{2}{3}} = \boxed{\frac{4}{3}}$

a) $(f \circ g)(x)$

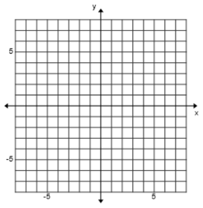
$f\left(8x^{-\frac{1}{2}}\right) = \left(8x^{-\frac{1}{2}}\right)^{\frac{2}{3}}$

$= 8^{\frac{2}{3}}\left(x^{-\frac{1}{2}}\right)^{\frac{2}{3}}$

$= \boxed{4x^{-\frac{1}{3}}}$

Test Review

Determine the domain and range based on the graph below.



Domain: ___________

Range: ___________

Inverse Function – Change x and y

Or a reflection over y = x.

$f^{-1}(x)$

Ex. 1) Algebraically find the inverse of f(x) = -2x + 5

$x = -2y + 5$

$-5 \qquad -5$

$x - 5 = -2y$

$\frac{x-5}{-2} = \frac{-2y}{-2}$

$\frac{x}{-2} + \frac{5}{2} = y = f^{-1}(x)$

2) Find the inverse of $f(x) = \sqrt{2x - 5} + 6$

$x = \sqrt{2y - 5} + 6$

$-6 \qquad -6$

$(x-6)^2 = (\sqrt{2y-5})^2$

$(x-6)^2 = 2y - 5$

$+5 \qquad +5$

$\frac{(x-6)^2 + 5}{2} = \frac{2y}{2}$

$\frac{(x-6)^2}{2} + \frac{5}{2} = y = f^{-1}(x)$

Proving Inverses

① find the compositions f(g(x)) then g(f(x))

If they are, they will = x.

3) Prove f(x) = 2x - 4 and g(x) = $\frac{x+4}{2}$ are inverses

$f(g(x))$

$\cancel{2}\left(\frac{x+4}{\cancel{2}}\right) - 4$

$x + \cancel{4} - \cancel{4}$

x

$g(f(x))$

$\frac{(2x-4)+4}{2}$

$\frac{\cancel{2}x - \cancel{4} + \cancel{4}}{\cancel{2}}$

x

Yes, they are inverses of each other b/c f(g(x)) = g(f(x)) = x.

4) Prove that f(x) and g(x) are inverses algebraically.

$f(x) = \sqrt[3]{x-2} - 1$

$g(x) = \sqrt[3]{x} - 2$

Worksheet EVENS only!!

2) $f(x) = -5x - 11$

4) $f(x) = \frac{1}{2}x + 7$

6) $f(x) = \sqrt{x-4}$

10) $f(x) = 27(x-1)^3$

8) $f(x) = 4(x+8)^2$

12) $g(x) = -\frac{1}{3}x - \frac{5}{3}$
$f(x) = \frac{-x-2}{2}$

14) $f(x) = \frac{8+7x}{4}$
$g(x) = \frac{4x-8}{7}$

16) $f(x) = \sqrt[3]{x-3} - 2$
$g(x) = \sqrt[3]{x} - 1$

18) $h(n) = -\frac{2}{n+1} + 2$
$f(n) = \frac{2}{-n+2} - 1$