Lesson 1.12 Objective: SWBAT determine the binomial expansion.

Kickoff

Find each of the following:

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2+2xy+y^2$

$(x+y)^3 =$ $(x+y)(x+y)(x+y)$

$(x+y)(x^2+2xy+y^2)$

$x^3 + \boxed{2x^2y} + xy^2 + \boxed{yx^2} + 2xy^2 + y^3$

$x^3 + 3x^2y^1 + 3x^1y^2 + y^3$

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1) $f(x) = 15x - 1$

$x = 15y - 1$

$x + 1 = 15y$

$\frac{x+1}{15} = y = f^{-1}(x)$

$\frac{x}{15} + \frac{1}{15} = f^{-1}(x)$

2) $f(x) = -5x - 11$

$x = -5y - 11$

$x + 11 = -5y$

$\frac{x+11}{-5} = y = f^{-1}(x)$

$\frac{x}{-5} - \frac{11}{5} = f^{-1}(x)$

3) $f(x) = 2x - 10$

$x = 2y - 10$

$x + 10 = 2y$

$\frac{x+10}{2} = y = f^{-1}(x)$

$\frac{x}{2} + 5 = f^{-1}(x)$

4) $f(x) = \frac{1}{2}x + 7$

$x = \frac{1}{2}y + 7$

$x - 7 = \frac{1}{2}y$

$2x - 14 = y = f^{-1}(x)$

$\frac{x-7}{\frac{1}{2}} = 2(x-7)$

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5) $f(x) = (x-2)^2$

$x = (y-2)^2$

$\pm\sqrt{x} = y - 2$

$\pm\sqrt{x} + 2 = y = f^{-1}(x)$

6) $f(x) = \sqrt{x} - 4$

$(x)^2 = (\sqrt{y} - 4)^2$

$x^2 = y - 4$

$x^2 + 4 = y$

$x^2 + 4 = f^{-1}(x)$

7) $f(x) = -4x^2 - 10$

$x = -4y^2 - 10$

$x + 10 = -4y^2$

$\frac{x+10}{-4} = \frac{-4y^2}{-4}$

$\pm\sqrt{\frac{x+10}{-4}} = \sqrt{y^2}$

$\pm\sqrt{\frac{x+10}{-4}} = f^{-1}(x)$

8) $f(x) = 4(x+8)^2$

$\frac{x}{4} = \frac{4(y+8)^2}{4}$

$\pm\sqrt{\frac{x}{4}} = \sqrt{(y+8)^2}$

$\pm\frac{\sqrt{x}}{2} = y + 8$

$\pm\frac{\sqrt{x}}{2} - 8 = f^{-1}(x)$

$\sqrt{\frac{x+10}{-4}} = \sqrt{y^2}$

$\pm\sqrt{\frac{x+10}{-4}}$

$f(x) = (x-2)^2$

$\pm\sqrt{x} = \sqrt{(y-2)^2}$

$\pm\sqrt{x} = y - 2$

$+2 \quad +2$

$\pm\sqrt{x} + 2 = f^{-1}(x)$

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9) $f(x) = \sqrt[3]{2x+7}$

$(x)^3 = (\sqrt[3]{2y+7})^3$

$x^3 = 2y + 7$

$x^3 - 7 = 2y$

$\frac{x^3-7}{2} = \frac{2y}{2}$

$\frac{x^3}{2} - \frac{7}{2} = f^{-1}(x)$

10) $f(x) = 27(x-1)^3$

$\frac{x}{27} = \frac{27(y-1)^3}{27}$

$\sqrt[3]{\frac{x}{27}} = \sqrt[3]{(y-1)^3}$

$\frac{\sqrt[3]{x}}{3} = y - 1$

$\frac{\sqrt[3]{x}}{3} + 1 = f^{-1}(x)$

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11) $g(x) = -\frac{1}{2}x - \frac{3}{2}$

$f(x) = -2x - 3$

$g(f(x))$

$-\frac{1}{2}(-2x-3) - \frac{3}{2}$

$x + \frac{3}{2} - \frac{3}{2}$

x

Yes, g(x) and f(x) are inverses of each other because $f(g(x)) = g(f(x)) = x$.

$f(g(x))$

$-2(-\frac{1}{2}x - \frac{3}{2}) - 3$

$x + 3 - 3$

x

12) $g(x) = -\frac{1}{3}x - \frac{5}{3}$

$f(x) = \frac{-x-2}{2}$

NO, g(x) and f(x) are not inverses of each other because $g(f(x)) \neq f(g(x)) \neq x$

$g(f(x))$

$-\frac{1}{3}(\frac{-x-2}{2}) - \frac{5}{3}$

$\frac{x+2}{6} - \frac{5(2)}{3(2)}$

$\frac{x+2}{6} - \frac{10}{6} = \frac{x-8}{6}$

$f(g(x))$

$\frac{-(-\frac{1}{3}x - \frac{5}{3}) - 2}{2}$

$\frac{\frac{1}{3}x + \frac{5}{3} - \frac{6}{3}}{2}$

$\frac{\frac{1}{3}x - \frac{1}{3}}{2} = \frac{1}{6}x - \frac{1}{6}$

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13) $f(x) = \frac{2}{3}x - \frac{2}{3}$

$g(x) = 1 + \frac{3}{2}x$

$f(g(x))$

$\frac{2}{3}\left(1 + \frac{3}{2}x\right) - \frac{2}{3}$

$\cancel{\frac{2}{3}} + x - \cancel{\frac{2}{3}}$

x

yes, g(x) and f(x) are inverses of each other because $f(g(x)) = g(f(x)) = x$.

$g(f(x))$

$1 + \frac{3}{2}\left(\frac{2}{3}x - \frac{2}{3}\right)$

$\cancel{1} + x - \cancel{1}$

x

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14) $f(x) = \frac{8+7x}{4}$

$g(x) = \frac{4x-8}{7}$

$f(g(x))$

$\frac{8 + \cancel{7}\left(\frac{4x-8}{\cancel{7}}\right)}{4}$

$\frac{\cancel{8} + 4x - \cancel{8}}{4}$

$\frac{4x}{4} = x$

yes, f(x) and g(x) are inverses of eachother because $f(g(x)) = g(f(x)) = x$.

$g(f(x))$

$\frac{4\left(\frac{8+7x}{4}\right) - 8}{7}$

$\frac{\cancel{8} + 7x - \cancel{8}}{7}$

$\frac{7x}{7} = x$

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15) $f(x) = \frac{-2x-6}{5}$

$g(x) = \frac{-6-5x}{2}$

$f(g(x))$

$\frac{-\cancel{2}\left(\frac{-6-5x}{\cancel{2}}\right) - 6}{5}$

$\frac{\cancel{6} + 5x - \cancel{6}}{5}$

$\frac{5x}{\cancel{5}} = x$

yes, f(x) and g(x) are inverses of eachother because $f(g(x)) = g(f(x)) = x$

$g(f(x))$

$\frac{-6 - \cancel{5}\left(\frac{-2x-6}{\cancel{5}}\right)}{2}$

$\frac{-\cancel{6} + 2x + \cancel{6}}{2}$

$\frac{2x}{2} = x$

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16) $f(x) = \sqrt[3]{x-3} - 2$

$g(x) = \sqrt[3]{x} - 1$

$f(g(x))$

$\sqrt[3]{(\sqrt[3]{x} - 1) - 3} - 2$

No, f(x) and g(x) are not inverses of each other because $g(f(x))$ $f(g(x)) \neq x$.

$g(f(x))$

$\sqrt[3]{(\sqrt[3]{x-3} - 2)} - 1$

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17) $f(x) = (x+3)^5 - 3$

$g(x) = \sqrt[5]{x+3} - 1$

$f(g(x))$

$((\sqrt[5]{x+3} - \cancel{1}) + \cancel{1})^5 - 3$

$(\sqrt[5]{x+3})^5 - 3$

$x + 3 - 3$

x

yes, f(x) and g(x) are inverses because $f(g(x)) = g(f(x)) = x$.

$g(f(x))$

$\sqrt[5]{((x+1)^5 - \cancel{3} + \cancel{3})} - 1$

$\sqrt[5]{(x+1)^5} - 1$

$x + \cancel{1} - \cancel{1}$

x

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18) $h(n) = -\frac{2}{n+1} + 2$

$f(n) = \frac{2}{-n+2} - 1$

$f(h(n))$

$\frac{2}{-\left(\frac{-2}{n+1} + \cancel{2}\right) + 2} - 1$

$\frac{2}{\frac{2}{n+1} - \cancel{2} + \cancel{2}} - 1$

$\frac{2}{\frac{2}{n+1}} - 1$

$\cancel{2} \cdot \frac{n+1}{\cancel{2}} - 1$

$n + \cancel{1} - \cancel{1} = n$

yes, f(x) and g(x) are inverses because $f(g(x)) = g(f(x)) = x$

$h(f(n))$

$-\frac{2}{\left(\frac{2}{-n+2} - 1\right) + 1} + 2$

$-\frac{2}{\frac{2}{-n+2} - \cancel{1} + \cancel{1}} + 2$

$\frac{-2}{\frac{2}{-n+2}} + 2$

$-\cancel{2} \cdot \frac{-n+2}{\cancel{2}} + 2$

$-1(-n+2) + 2$

$n - \cancel{2} + \cancel{2}$

n

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Pascal's Triangle

*coefficients

1 exp 0

1 1 exp 1

1 2 1 exp 2

1 3 3 1 exp 3

1 4 6 4 1 exp 4

1 5 10 10 5 1 exp 5

1 6 15 20 15 6 1

$_0C_0$

$_1C_1$ $_1C_0$

$_2C_2$ $_2C_1$ $_2C_0$

$_3C_3$ $\boxed{_3C_2}$ $_3C_1$ $_3C_0$

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Find each of the following using Pascal's Triangle

5) $(x+y)^4$

$1x^4 + 4x^3y^1 + 6x^2y^2 + 4xy^3 + 1y^4$

6) $\cancel{(m+2n)^{15}}$

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Binomial Expansion: $nCr\, a^{n-r}b^r$ for $(p-q)^n$

What do those symbols mean?

nCr - Combination without order.
*coefficients

b^r - 2nd term ex: $(x+y)^5$

a^{n-r} - 1st term ex: $(x+y)^5$

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Example:

1) $(1-a)^5$ nCr $n=5$ $r=5,4,3,2,1,0$

$_5C_5(1)^5(-a)^0 + {_5C_4}(1)^4(-a)^1 + {_5C_3}(1)^3(-a)^2 + {_5C_2}(1)^2(-a)^3 + {_5C_1}(1)^1(-a)^4 + {_5C_0}(1)^0(-a)^5$

$(1)(1)^5(1) + (5)(1)^4(-a) + (10)(1)^3(a^2) + (10)(1)^2(-a^3) + (5)(1)^1(a^4) + (1)(1)(-a^5)$

$\boxed{1-5a+10a^2-10a^3+5a^4-a^5}$

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2) $(2y^4-1)^6$ $n=6$ $r=6,5,4,3,2,1,0$

$(1)\,{_6C_6}(2y^4)^6(-1)^0 + (6)\,{_6C_5}(2y^4)^5(-1)^1 + (15)\,{_6C_4}(2y^4)^4(-1)^2 + (20)\,{_6C_3}(2y^4)^3(-1)^3 + (15)\,{_6C_2}(2y^4)^2(-1)^4 + (6)\,{_6C_1}(2y^4)(-1)^5 + (1)\,{_6C_0}(2y^4)^0(-1)^6$

$(1)(2^6)(-1)^0y^{24} + (6)(2^5)(-1)^1y^{20} + (15)(2^4)(-1)^2y^{16} + (20)(2^3)(-1)^3y^{12} + (15)(2^2)(-1)^4y^8 + (6)(2)(-1)^5y^4 + (1)(1)(-1)^6$

$64y^{24}-192y^{20}+240y^{16}-160y^{12}+60y^8-12y^4-1$

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3) $(2x+3)^5$ $n=5$ $r=5,4,3,2,1,0$

$_5C_5(2x)^5(3)^0 = (1)(32x^5)(1) = 32x^5$

$_5C_4(2x)^4(3)^1 = (5)(16x^4)(3) = 240x^4$

$_5C_3(2x)^3(3)^2 = (10)(8x^3)(9) = 720x^3$

$_5C_2(2x)^2(3)^3 = (10)(4x^2)(27) = 1080x^2$

$_5C_1(2x)^1(3)^4 = (5)(2x)(81) = 810x$

$_5C_0(2x)^0(3)^5 = (1)(1)(243) = 243$

$32x^5+240x^4+720x^3+1080x^2+810x+243$

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4) $(2x^3 - y)^5$ $n=5$

$r = 5, 4, 3, 2, 1, 0$

$_5C_5 (2x^3)^5 (-y)^0 = (1)(32x^{15})(1) = 32x^{15}$

$_5C_4 (2x^3)^4 (-y)^1 = (5)(16x^{12})(-y) = -80x^{12}y$

$_5C_3 (2x^3)^3 (-y)^2 = (10)(8x^9)(y^2) = 80x^9y^2$

$_5C_2 (2x^3)^2 (-y)^3 = (10)(4x^6)(-y^3) = -40x^6y^3$

$_5C_1 (2x^3)^1 (-y)^4 = (5)(2x^3)(y^4) = 10x^3y^4$

$_5C_0 (2x^3)^0 (-y)^5 = (1)(1)(-y^5) = -y^5$

$32x^{15} - 80x^{12}y - 40x^6y^3 + 10x^3y^4 - y^5$

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5) $(2m - 3n)^4$

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6) $(m^3 + 2n)^6$

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7) $(a^3 - b^2)^7$

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