Binomial Expansion and nth term

Kickoff

1) Using Pascal's Triangle, expand the following: $(a+b)^6$

$1a^6 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$

2) State if the following functions are inverses. (algebraically)

$f(x) = \frac{8+7x}{4}$

$g(x) = \frac{4x-8}{7}$

$f(g(x))$

$\frac{8+7\left(\frac{4x-8}{7}\right)}{4}$

$\frac{8+\cancel{7}\frac{(4x-8)}{\cancel{7}}}{4}$

$\frac{\cancel{8}+4x-\cancel{8}}{4}$

$\frac{\cancel{4}x}{\cancel{4}} = x$

$g(f(x))$

$\frac{\cancel{4}\left(\frac{8+7x}{\cancel{4}}\right)-8}{7}$

$\frac{\cancel{8}+7x-\cancel{8}}{7}$

$\frac{\cancel{7}x}{\cancel{7}} = x$

Yes b/c $f(g(x)) = g(f(x)) = x$.

Lesson1.13- SWBAT determine the binomial expansions nth term.

Kickoff

Complete question 5 in the packet from yesterday (write it vertically!)

ex: $n=4$ $r=4,3,2,1,0$

${}_4C_4(2m)^4(-3n)^0 = (1)(16m^4)(1) = 16m^4$

${}_4C_3(2m)^3(-3n)^1 =$

⋮

5) $(2m-3n)^4$

${}_4C_4(2m)^4(-3n)^0 = (1)(2^4m^4)(1) = 16m^4$

${}_4C_3(2m)^3(-3n)^1 = (4)(2^3m^3)(-3n) = -96m^3n$

${}_4C_2(2m)^2(-3n)^2 = (6)(2^2m^2)(-3^2n^2) = 216m^2n^2$

${}_4C_1(2m)^1(-3n)^3 = (4)(2m)(-3^3n^3) = -216mn^3$

${}_4C_0(2m)^0(-3n)^4 = (1)(1)(-3^4n^4) = 81n^4$

$16m^4 - 96m^3n + 216m^2n^2 - 216mn^3 + 81n^4$

2) $(2y^4-1)^6$ $n=6$ $r=6,5,4,3,2,1,0$

$a = 2y^4$, $b = -1$

$(1)\,{}_6C_6(2y^4)^6(-1)^0 + (6)\,{}_6C_5(2y^4)^5(-1)^1 + (15)\,{}_6C_4(2y^4)^4(-1)^2 + (20)\,{}_6C_3(2y^4)^3(-1)^3 + (15)\,{}_6C_2(2y^4)^2(-1)^4 + (6)\,{}_6C_1(2y^4)^1(-1)^5 + (1)\,{}_6C_0(2y^4)^0(-1)^6$

2^6y^{24}, 2^5y^{20}, 2^4y^{16}, 2^3y^{12}, 2^2y^8, $2y^4$

$(1)(2^6)(-1)^0y^{24} + (6)(2^5)(-1)^1y^{20} + (15)(2^4)(-1)^2y^{16} + (20)(2^3)(-1)^3y^{12} + (15)(2^2)(-1)^4y^8 + (6)(2)(-1)^5y^4 + (1)(1)(-1)^6$

$64y^{24} - 192y^{20} + 240y^{16} - 160y^{12} + 60y^8 - 12y^4 + 1$

3) $(2x+3)^5$ $n=5$ $r=5,4,3,2,1,0$

${}_5C_5(2x)^5(3)^0 = (1)(32x^5)(1) = 32x^5$

${}_5C_4(2x)^4(3)^1 = (5)(16x^4)(3) = 240x^4$

${}_5C_3(2x)^3(3)^2 = (10)(8x^3)(9) = 720x^3$

${}_5C_2(2x)^2(3)^3 = (10)(4x^2)(27) = 1080x^2$

${}_5C_1(2x)^1(3)^4 = (5)(2x)(81) = 810x$

${}_5C_0(2x)^0(3)^5 = (1)(1)(243) = 243$

$32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$

4) $(2x^3-y)^5$ $n=5$ $r=5,4,3,2,1,0$

${}_5C_5(2x^3)^5(-y)^0 = (1)(32x^{15})(1) = 32x^{15}$

${}_5C_4(2x^3)^4(-y)^1 = (5)(16x^{12})(-y) = -80x^{12}y$

${}_5C_3(2x^3)^3(-y)^2 = (10)(8x^9)(y^2) = 80x^9y^2$

${}_5C_2(2x^3)^2(-y)^3 = (10)(4x^6)(-y^3) = -40x^6y^3$

${}_5C_1(2x^3)^1(-y)^4 = (5)(2x^3)(y^4) = 10x^3y^4$

${}_5C_0(2x^3)^0(-y)^5 = (1)(1)(-y^5) = -y^5$

$32x^{15} - 80x^{12}y - 40x^6y^3 + 10x^3y^4 - y^5$

Finding nth terms

Steps

1) Write all possible r values
$r = 4, 3, 2, 1, 0$
2) Count to the term you need. (2nd term)
3) Use r value to complete the formula/pattern
4) Multiply the coefficients/Simplify.

1) Find the coefficient of the y^2 in the expansion of $(2y^2 - 1)^6$

$${}_6C_1 (2y^2)^1 (-1)^5 = (6)(2y^2)(-1) = \boxed{-12}\, y^2$$

2) Find the coefficient of y^8x^3 in the expansion of $(y^4 - 3x)^5$

$${}_5C_2 (y^4)^2 (-3x)^3 = (10)(y^8)(-27x^3)$$
$$= \boxed{-270}\, y^8x^3$$

3) Find the coefficient of x^2y^3 in the expansion of $(x^2 - 3y)^4$

$${}_4C_1 (x^2)^1 (-3y)^3 = (4)(x^2)(-27y^3)$$
$$\boxed{-108}\, x^2y^3$$

4) Find the 4th term in the expansion of $(1 - 5x^3)^3$

$r = 3, 2, 1, 0$
$n = 3$
$-5^3 x^9$
$${}_3C_0 (1)^0 (-5x^3)^3 = (1)(1)(-125x^9)$$
$$= -125x^9$$

5) Find the 5th term in the expansion of $(1 - 4m^2)^4$

$r = 4, 3, 2, 1, 0$
$n = 4$
$${}_4C_0 (1)^0 (-4m^2)^4 = (1)(1)(256m^8)$$
$$256m^8$$

6) Find the 2nd term in the expansion of $(1 - 3y^4)^4$

7) Find the 3rd term in the expansion of $(2x + 3)^5$

8) Find the seventh term in the expansion of $(4x - 6y)^9$.

9) Find the sixth term in the expansion of $(4x - 5y)^8$.