

Lesson 1.4 Objective: SWBAT determine the domain and range of a function graphically.

Kickoff

On the post it on your desk (put your name on it), evaluate the difference quotient with  $f(x) = x^2 - 5x$

$$\frac{f(x+h) - f(x)}{h} \quad f(x+h) = (x+h)^2 - 5(x+h)$$

$$\frac{[(x+h)^2 - 5(x+h)] - [x^2 - 5x]}{h}$$

$$\frac{x^2 + xh + hx + h^2 - 5x - 5h - x^2 + 5x}{h}$$

$$\frac{2hx + h^2 - 5h}{h} = 2x + h - 5$$

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80)  $\frac{3(x+h) - 1 - [3x - 1]}{h}$

$$\frac{3x + 3h - 1 - 3x + 1}{h} = \frac{3h}{h} = 3$$

82)  $\frac{f(5+h) - f(5)}{h} = \frac{[5(5+h) - (5+h)^2] - [5(5) - 5^2]}{h}$

$$\frac{[25 + 5h - (25 + 10h + h^2)] - [25 - 25]}{h}$$

$$\frac{25 + 5h - 25 - 10h - h^2 - 25 + 25}{h}$$

$$\frac{-5h - h^2}{h} = -5 - h$$

$\frac{f(x+h) - f(x)}{h}$  More practice

$f(x) = x^2 + 2$

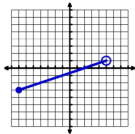
$$\frac{[(x+h)^2 + 2] - [x^2 + 2]}{h}$$

$g(x) = 2x^2 - x$

EXAMPLES OF DOMAINS AND RANGES FROM GRAPHS

Important notes about Domains and Ranges from Graphs:

- Remember that **domain** refers to the x-values that are represented in a problem and **range** refers to the y-values that are represented in a problem.
- Sometimes it isn't possible to list all the values that x or y can be because the graph is **continuous** and made up of an infinite number of points, like a line, a ray, or even a segment.
- In a continuous graph, to determine the **domain**, you should focus on looking **left to right of the graph**.
- In a continuous graph, to determine the **range**, you should focus on looking **bottom to top of the graph**.
- We use **interval notation** to help us describe the domain and range for graphs that represent continuous situations.
- Please review the following information to help you describe the domain and range for three different types of continuous graphs.



Domain:  $-7 \leq x < 5$

- Notice that this graph has two endpoints, so the graph starts and stops and the **domain covers all x-values between the two endpoints** which makes it a continuous graph.
- Since the left and right endpoints are at  $(-7, -3)$  and  $(5, 1)$ , the graph covers all **x-values between the x-values of -7 and -3**.
- Notice that the first endpoint is a closed circle so it includes that point, but the second endpoint is an open circle, so it does not include that point.
- Therefore, the graph covers **all x-values  $\geq -7$  AND all x-values  $< 5$**  - we write that in interval notation as  $[-7 \leq x < 5]$

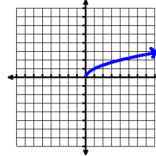
Range:  $-3 \leq y < 1$

- Notice that this graph has two endpoints, so the graph starts and stops and the **range covers all y-values between the two endpoints** which makes it a continuous graph.
- Since the bottom and top endpoints are at  $(-7, -3)$  and  $(5, 1)$ , the graph covers all **y-values between the y-values of -3 and 1**.
- Notice that the first endpoint is a closed circle so it includes that point, but the second endpoint is an open circle, so it does not include that point.
- Therefore, the graph covers **all y-values  $\geq -3$  AND all y-values  $< 1$**  - we write that in interval notation as  $[-3 \leq y < 1]$

Important Note:

- To find the domain for a graph with two endpoints, always identify the x-values of the point farthest to the left and the point farthest to the right.
- For the range, you want the y-values of the lowest point and the highest point.

Example 2 - a continuous graph with only one endpoint (so continues forever in the other direction)



Domain:  $x \geq 0$  (remember to focus on left to right of the graph for domain of a continuous graph):

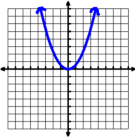
- Notice that this graph has one endpoint at  $(0, 0)$  and an arrow to the right indicating that it continues forever in the positive x direction.
- Therefore, this graph covers all x-values that are greater than or equal to 0 - there is no stopping point on the right side of the graph.
- We write the domain in interval notation as  $\{x \geq 0\}$ .

Range:  $y \geq 0$  (remember to focus on bottom to top of the graph for range of a continuous graph):

- Notice that this graph has one endpoint at  $(0, 0)$  and an arrow pointing up indicating that it continues forever in the positive y direction.
- Therefore, this graph covers all y-values that are greater than or equal to 0 - there is no stopping point on the upper side of the graph.
- We write the range in interval notation as  $\{y \geq 0\}$ .

Note: If the arrow were pointing to the left, the domain would be  $\leq$  the x-value. If the arrow were pointing down, the range would be  $\leq$  the y-value.

**Example 3 – a continuous graph that has two arrows:**



**Domain:  $\{x = \text{all real numbers}\}$**  (remember to focus on left to right of the graph to determine the domain for a continuous graph)

- Notice that this graph has an **arrow on the left side** of the graph and an **arrow on the right side** of the graph.
- This indicates that the graph **continues forever in the left direction and forever in the right direction**.
- This means that the graph covers **all possible x-values** – we call that all real numbers in algebra.
- Therefore, we can write the domain in interval notation as:  $\{x = \text{all real numbers}\}$ .

**Range:  $\{y \geq 0\}$**  (remember to focus on bottom to top of the graph to determine the range of a continuous graph)

- Notice that the **graph's lowest point is at  $(0, 0)$**  (the bottom of the parabola) – indicating that the y-values start at 0.
- However, notice at the **top of the graph there are arrows pointing up** – this indicates the graph continues in the positive y direction forever.
- So, the graph covers **all y-values greater than or equal to 0**.
- We can write the range in interval notation as:  $\{y \geq 0\}$ .

Note: If one of the arrows were pointing up and one of the arrows were pointing down, then the range would be all real numbers.

_____ 1. Domain: $\{-4 \leq x \leq 4\}$ Range: $\{-4 \leq y \leq 4\}$ Function: NO	_____ 2. Domain: $\{-3 < x \leq 5\}$ Range: $\{y = -1\}$ Function: YES	_____ 3. Domain: $\{-4 \leq x \leq 2\}$ Range: $\{-2 \leq y \leq 4\}$ Function: YES
_____ 4. Domain: $\{x > 0\}$ Range: $\{y = 4\}$ Function: YES	_____ 5. Domain: $\{-6 \leq x \leq 6\}$ Range: $\{0 \leq y \leq 6\}$ Function: YES	_____ 6. Domain: $\{x = -5\}$ Range: $\{-2 < y < 6\}$ Function: NO

_____ 7. Domain: $\{x \geq 0\}$ Range: $\{\text{all real numbers}\}$ Function: NO	_____ 8. Domain: $\{-3 \leq x \leq 4\}$ Range: $\{-2 \leq y \leq 4\}$ Function: NO	_____ 9. Domain: $\{\text{all real numbers}\}$ Range: $\{\text{all real numbers}\}$ Function: YES
_____ 10. Domain: $\{-7 \leq x < 5\}$ Range: $\{-3 \leq y < 1\}$ Function: YES	_____ 11. Domain: $\{\text{all real numbers}\}$ Range: $\{y \geq 0\}$ Function: YES	_____ 12. Domain: $\{-3 < x < 4\}$ Range: $\{0 \leq y \leq 5\}$ Function: YES

_____ 13. Domain: $\{-6 \leq x \leq 3\}$ Range: $\{-6 \leq y \leq -1\}$ Function: YES	_____ 14. Domain: $\{0 \leq x < 5\}$ Range: $\{0 \leq y < 7\}$ Function: YES	_____ 15. Domain: $\{-5 \leq x < 0\}$ Range: $\{-5 < y \leq -1\}$ Function: YES
_____ 16. Domain: $\{-6 \leq x \leq 3\}$ Range: $\{-5 \leq y \leq -1\}$ Function: YES	_____ 17. Domain: $\{0 \leq x \leq 6\}$ Range: $\{0 \leq y \leq 7\}$ Function: YES	_____ 18. Domain: $\{-4 \leq x \leq 7\}$ Range: $\{-7 \leq y \leq -2\}$ Function: NO

_____ 19. Domain: $\{x \leq 0\}$ Range: $\{y \geq 0\}$ Function: YES	_____ 20. Domain: $\{2 \leq x \leq 7\}$ Range: $\{1 \leq x \leq 6\}$ Function: NO	_____ 21. Domain: $\{0 \leq x \leq 4\}$ Range: $\{0 \leq y \leq 6\}$ Function: YES
_____ 22. Domain: $\{-4 < x < 5\}$ Range: $\{-2 \leq y < 5\}$ Function: YES	_____ 23. Domain: $\{x \leq 5\}$ Range: $\{y = 0\}$ Function: YES	_____ 24. Domain: $\{-7 < x < 0\}$ Range: $\{-3 < y < 4\}$ Function: YES