Name: ____________________________________ Date: _________________

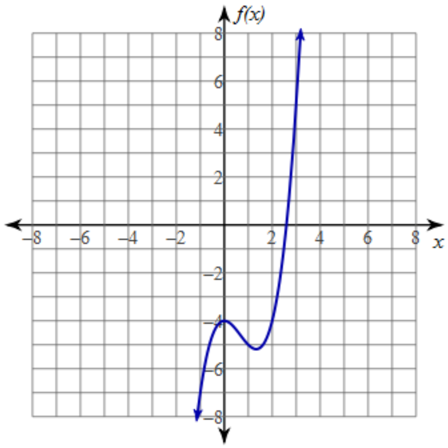
Unit: Limits **Video: Limits Algebraically at Horizontal Asymptotes**

<u>Polynomimal Functions</u>- Use the leading coefficient and the degree to determine the behavior as x approaches infinity or negative infinity.

Odd Degree

Positive Leading Coefficient

$$f(x) = x^3 - 2x^2 - 4$$

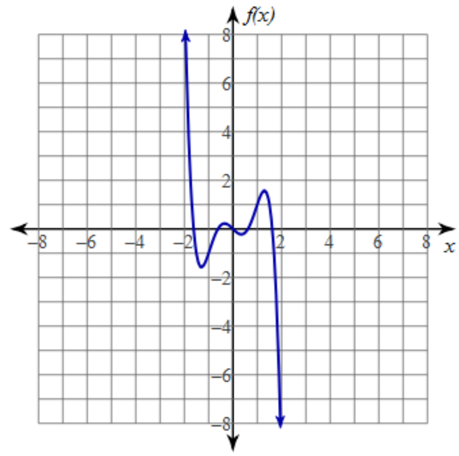


$$\lim_{x\to\infty}(x^3 - 2x^2 - 4) =$$

$$\lim_{x\to-\infty}(x^3 - 2x^2 - 4) =$$

Negative Leading Coefficient

$$f(x) = -x^5 + 3x^3 - x$$



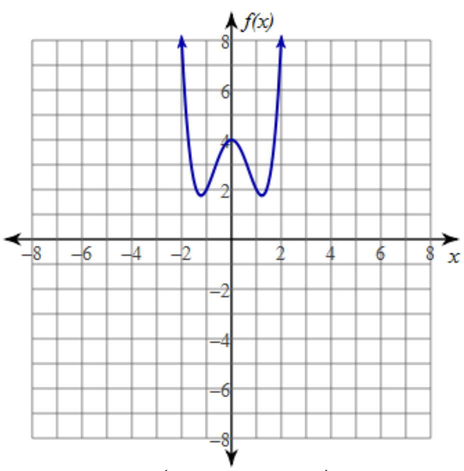
$$\lim_{x\to\infty}\left(-x^5 + 3x^3 - x\right) =$$

$$\lim_{x\to-\infty}\left(-x^5 + 3x^3 - x\right) =$$

Even Degree

Positive Leading Coefficient

$$f(x) = x^4 - 3x^2 + 4$$

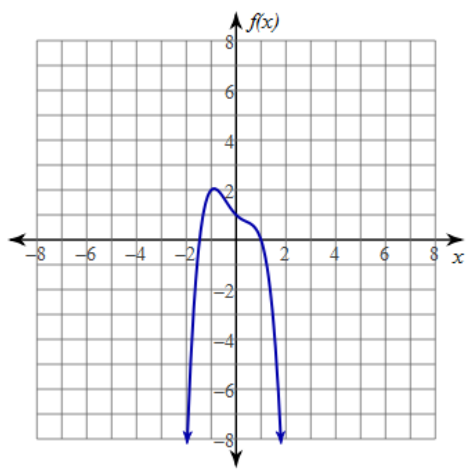


$$\lim_{x\to\infty}\left(x^4 - 3x^2 + 4\right) =$$

$$\lim_{x\to-\infty}\left(x^4 - 3x^2 + 4\right) =$$

Negative Leading Coefficient

$$f(x) = -x^4 + x^2 - x + 1$$



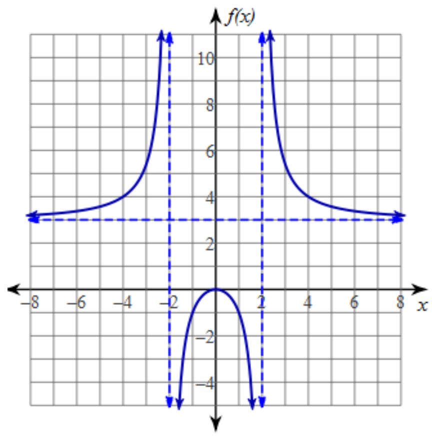
$$\lim_{x\to\infty}\left(-x^4 + x^2 - x + 1\right) =$$

$$\lim_{x\to-\infty}\left(-x^4 + x^2 - x + 1\right) =$$

Rational Functions - Use the Horizontal Asymptote Rules

***Rule #1**: If the degree of the numerator = degree of the denominator, then the horizontal asymptote is the ratio of the coefficients*

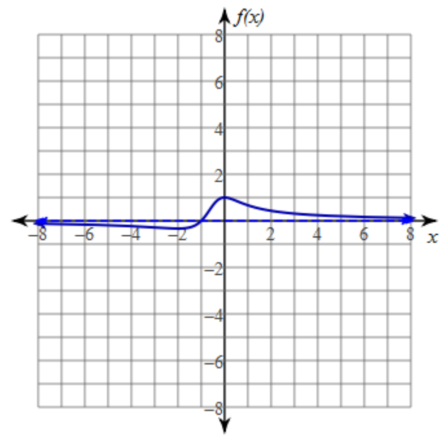
$$f(x)=\frac{3x^2}{x^2-4}$$



$$\lim_{x\to\infty}\left(\frac{3x^2}{x^2-4}\right)= \qquad \lim_{x\to-\infty}\left(\frac{3x^2}{x^2-4}\right)=$$

***Rule # 2**: If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is equal to y = 0*

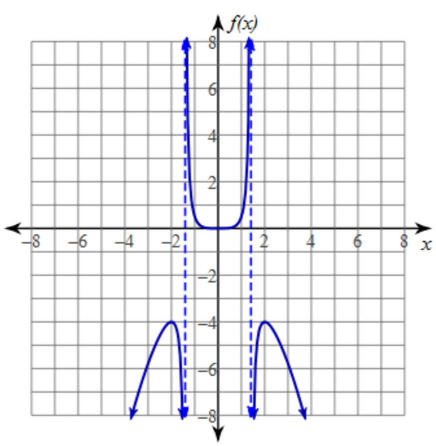
$$f(x)=\frac{x+1}{x^2+x+1}$$



$$\lim_{x\to\infty}\left(\frac{x+1}{x^2+x+1}\right)= \qquad \lim_{x\to-\infty}\left(\frac{x+1}{x^2+x+1}\right)=$$

***Rule #3**: If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asympote and the graph approaches either positive or negative infinity.*

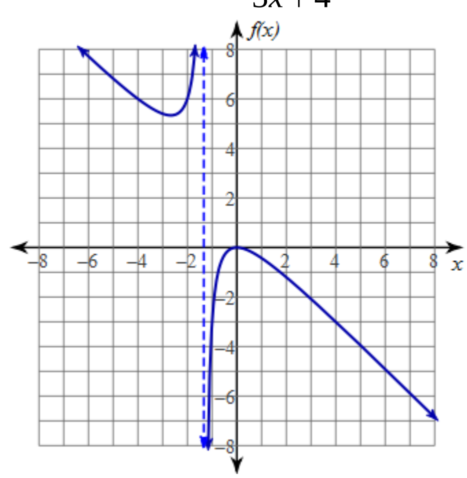
$$f(x)=-\frac{x^4}{2x^2-4}$$



$$\lim_{x\to\infty}\left(-\frac{x^4}{2x^2-4}\right)=$$

$$\lim_{x\to-\infty}\left(-\frac{x^4}{2x^2-4}\right)=$$

$$f(x)=-\frac{3x^2}{3x+4}$$



$$\lim_{x\to\infty}\left(-\frac{3x^2}{3x+4}\right)=$$

$$\lim_{x\to-\infty}\left(-\frac{3x^2}{3x+4}\right)=$$