Lesson 103 Objective: SWBAT understand and determine the derivative of a function.

Kickoff

1) Determine if the function is continuous at x = 2.

$$f(x)=\begin{cases}2x-10, & x<2\\ 0, & x\geq 2\end{cases}$$

① $f(2)=0$

② $\lim_{x\to 2}$ to exist?

2) Find the value of k that would make the following function continuous at x = 3.

$$f(x)=\begin{cases}x^2-1, & \text{if } x<3\\ 2kx, & \text{if } x\geq 3\end{cases}$$

$(3)^2-1=2k(3)$

$\frac{8}{6}=\frac{6k}{6}$

$\boxed{\frac{4}{3}=k}$

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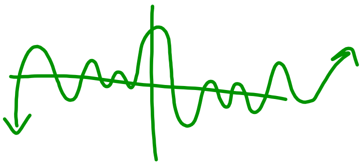
Continuous Functions

- when you can draw a graph without lifting up your pen



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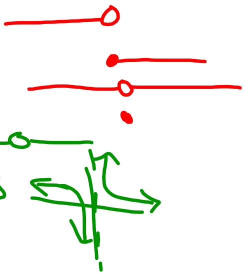
Discontinuous Functions

Types of Discontinuous Functions:

1) Jump
2) Removable
3) holes
4) asymptotes



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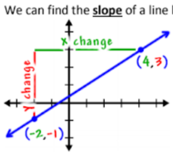
Derivatives are all about slopes!

We can find the **slope** of a line between any two points using the following formula

$$Slope=\frac{\Delta Y}{\Delta X}$$



$\frac{y_2-y_1}{x_2-x_1}$

$\frac{3--1}{4--2}=\frac{4}{6}=\frac{2}{3}$

derivative

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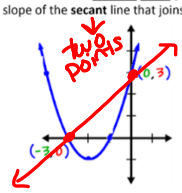
We can find the **average slope** (**average rate of change**) between two points on a curve by taking the slope of the **secant** line that joins the two points.



$\frac{3-0}{0--3}=\frac{3}{3}=1$

Average rate of change (derivative)

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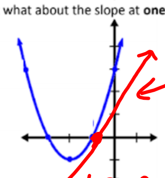
But what about the slope at **one** point on a curve? This is called the **instantaneous rate of change.**

one point!



slope is called derivative!

tangent line.

Limit Definition of the Derivative

$$f'(x)=\lim_{h\to 0}\boxed{\frac{f(x+h)-f(x)}{h}}$$

Notations used to represent derivative: $f'(x)$ or $\frac{dy}{dx}$

$f'(x)$

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Example #1: Using the limit definition, find the derivative, $f'(x)$ of $f(x) = -3x + 8$.

$$\lim_{h\to 0} \frac{\boxed{-3(x+h)+8} - \boxed{-3x+8}}{h} \qquad m = -3$$

$$\frac{\cancel{-3x} - 3h \cancel{+8} \cancel{+3x} \cancel{-8}}{h}$$

$$\frac{-3\cancel{h}}{\cancel{h}}$$

$$\lim_{h\to 0} -3 = \boxed{-3}$$

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Example #2: Using the limit definition find the derivative, $f'(x)$ of $f(x) = 2x^2 - x + 5$.

$$\lim_{h\to 0} \frac{\boxed{2(x+h)^2 - (x+h)+5} - \boxed{2x^2 - x + 5}}{h}$$

$$\frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-x} - h \cancel{+5} \cancel{-2x^2} \cancel{+x} \cancel{-5}}{h}$$

$$\frac{4xh + 2h^2 - h}{h}$$

$$\lim_{h\to 0} 4x + 2h - 1 = f'(x)$$

$$4x + 2(0) - 1 = 4x - 1$$

What does $f'(x) = 4x - 1$ mean?

It means that for the function $f(x) = 2x^2 - x + 5$, the derivative at any point is 4x -1

So, for example, when x = 4, the slope of the tangent line is

$$f'(4) = 4(4) - 1 = \boxed{15}$$

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Example #3: Using the limit definition of the derivative, find $\frac{dy}{dx}$ if, $f(x) = \frac{x+1}{x}$.

$$\lim_{h\to 0} \frac{\frac{(x)(x+h)+1}{(x)\,x+h} - \frac{x+1}{x}\frac{(x+h)}{(x+h)}}{h}$$

$$\frac{\frac{\cancel{x^2} + \cancel{xh} + \cancel{x} - \boxed{\cancel{x^2} + \cancel{xh} + \cancel{x} + h}}{x(x+h)}}{h} \qquad -\cancel{x^2} - \cancel{xh} - \cancel{x} - h$$

What is the derivative at x = 4?

$$f'(4) = -\frac{1}{4^2} = \boxed{-\frac{1}{16}}$$

$$\frac{\frac{-h}{x(x+h)}}{\frac{h}{1}} = \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x)}$$

$$f'(x) = \frac{-1}{x^2}$$

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Example 4: Using the limit definition of the derivative, find $f'(x)$ of $f(x) = \sqrt{x+5}$.

$$\lim_{h\to 0} \frac{\sqrt{x+h+5} - \sqrt{x+5}}{h} \cdot \frac{(\sqrt{x+h+5} + \sqrt{x+5})}{(\sqrt{x+h+5} + \sqrt{x+5})}$$

$$\frac{(\cancel{x} + h \cancel{+5}) - \cancel{(x+5)}}{h(\sqrt{x+h+5} + \sqrt{x+5})} \qquad \cancel{-x} \cancel{-5}$$

$$\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+5} + \sqrt{x+5})}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{x+\cancel{h}+5} + \sqrt{x+5}}$$

$$\frac{1}{\sqrt{x+5} + \sqrt{x+5}} = \frac{1}{2\sqrt{x+5}}$$

What is the derivative at (-2, 3)?

$$\frac{1}{2\sqrt{-2+5}}$$

$$\frac{1}{2\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{\frac{\sqrt{3}}{6}}$$

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