Objective: SWBAT use derivatives to find tangent and normal lines.

Kickoff

Find the derivative of each of the following:

1) $f(x) = 2x^2 - 1$

2) $f(x) = \sqrt{4x-5}$

3) $f(x) = -\frac{2}{2x-1}$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

① $\lim_{h\to 0} \frac{[2(x+h)^2-1]-[2x^2-1]}{h}$

$\lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2\cancel{-1}\cancel{-2x^2}\cancel{+1}}{h}$

$\frac{4x\cancel{h}+2h^{\cancel{2}}}{\cancel{h}}$

$\lim_{h\to 0} 4x+2h = 4x+2(0)$

$4x$

② $\sqrt{4x-5}$

$\lim_{h\to 0} \frac{\sqrt{4(x+h)-5}-\sqrt{4x-5}}{h}$

$(\sqrt{4x+4h-5}+\sqrt{4x-5})$ $\frac{\sqrt{4x+4h-5}-\sqrt{4x-5}}{h}$

$\frac{(\cancel{4x}+4h\cancel{-5})\cancel{-(4x-5)}}{h(\sqrt{4x+4h-5}+\sqrt{4x-5})}$

$\frac{4\cancel{h}}{\cancel{h}(\sqrt{4x+4h-5}+\sqrt{4x-5})}$

$\lim_{h\to 0} \frac{4}{\sqrt{4x+\cancel{4h}-5}+\sqrt{4x-5}}$

$\frac{4}{\sqrt{4x-5}+\sqrt{4x-5}} = \frac{4}{2\sqrt{4x-5}} = \frac{2}{\sqrt{4x-5}}$

③ $\frac{-2}{2x-1}$

$\lim_{h\to 0} \frac{\left(\frac{-2}{2(x+h)-1}\right)-\left(\frac{-2}{2x-1}\right)}{h}$

$\frac{\frac{-2}{2x+2h-1}\cdot\frac{(2x-1)}{(2x-1)}+\frac{2}{2x-1}\cdot\frac{(2x+2h-1)}{(2x+2h-1)}}{h}$

$\frac{\frac{\cancel{-4x}\cancel{+2}+\cancel{4x}+4h\cancel{-2}}{(2x+2h-1)(2x-1)}}{h} = \frac{\frac{4h}{(2x+2h-1)(2x-1)}}{\frac{h}{1}}$

$\frac{4\cancel{h}}{(2x+2h-1)(2x-1)}\cdot\frac{1}{\cancel{h}} = \frac{4}{(2x+\cancel{2h}-1)(2x-1)}$

$\frac{4}{(2x-1)(2x-1)} = \frac{4}{(2x-1)^2}$

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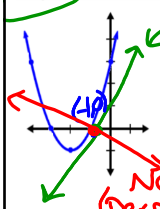
Tangent and Normal Lines at a Point



Slope of the tangent line = derivative

Normal Line (perpendicular) *opposite reciprocals!

Equation of a Line in Point- Slope Form

$y - y_1 = m(x - x_1)$

Point! derivative Points!

Leave x and y!

Do not plug anything into those variables

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Example #1: Using $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, find the following for $f(x) = 4x^2 - x + 1$

a) $f'(x)$

b) The equation of the tangent line and the normal line at (3,1)

a) $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \frac{[4(x+h)^2-(x+h)+1]-[4x^2-x+1]}{h}$

$\frac{\cancel{4x^2}+8xh+4h^2\cancel{-x}-h\cancel{+1}\cancel{-4x^2}\cancel{+x}\cancel{-1}}{h}$

$\frac{8x\cancel{h}+4h^{\cancel{2}}-\cancel{h}}{\cancel{h}}$

$\lim_{h\to 0} 8x+4h-1 = 8x-1$

$f'(x)$

b) $(3,1)$ x_1, y_1

Tangent

$y-1 = 23(x-3)$

$f'(x) = 8x-1$

$f'(3) = 8(3)-1 = 23$

Normal

$y-1 = -\frac{1}{23}(x-3)$

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Concept Check: Using $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, find the following for $f(x) = -x^2 - 2x + 7$

a) $f'(x)$

b) The equation of the tangent line and the normal line at x = -2

a) $f'(x) = -2x-2$

b) $f'(-2) = 4-2 = 2$

Tangent

$y-7 = 2(x+2)$

Normal

$y-7 = -\frac{1}{2}(x+2)$

$f(-2) = -(-2)^2 - 2(-2) + 7 = 7$

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