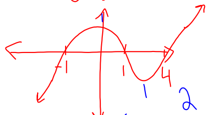
Lesson 2.11- Objective: SWBAT apply the remainder and factor theorem with polynomials.

Take out your homework and answer the following:
1) Divide $2x^2 - 4x+7$ by $x-1$ using synthetic division.
2) Graph the following polynomial WITHOUT a calculator. $f(x)=x^3-4x^2-x+4$ (Be sure to state ALL necessary parts)

① 1 | 2 -4 7
↓ 2 -2
2 -2 | 5

$2x - 2 + \frac{5}{x-1}$

② $f(x)=(x^3-4x^2)(-x+4)$
$x^2(x-4) -1(x-4)$
$(x^2-1)(x-4)$
$(x+1)(x-1)(x-4)$
LC: 1
D: 3
$x\to\infty$ $f(x)\to\infty$
$x\to-\infty$ $f(x)\to-\infty$
$x+1=0$ $x-1=0$ $x-4=0$
$x=-1$ cross, $x=1$ cross, $x=4$ cross



(extrema)
* total mins/max is one less than your degree

③ $x+4$
④ x^2-x-3
⑤ $7x^3-3x^2+3$
⑥ $x^2 - \frac{4}{x-2}$
⑦ $2x^2-3x-3+\frac{2}{x+1}$
⑧ $6x^4-3x^3-2x^2+1-\frac{10}{x+4}$
⑨ $2x^2+4x+3-\frac{1}{x-2}$
⑩ $4x^2-32x+241-\frac{1911}{x+8}$

Remainder and Factor Theorem

Remainder Theorem- If you divide a polynomial by $(x-k)$ then the remainder is $f(k)$.

Example 1: $f(x) = 3x^3 + 8x^2 + 5x - 7$ find $f(-2)$ using synthetic division.

-2 | 3 8 5 -7
↓ -6 -4 -2
3 2 1 | -9

$f(-2)=3(-2)^3+8(-2)^2+5(-2)-7=-9$

Factor Theorem- A polynomial $f(x)$ has a factor of $(x-k)$ if $f(k)=0$ (there is no remainder!)

Example 2: Show that $(x-7)$ is a factor of $2x^2 - 11x - 21$

7 | 2 -11 -21
↓ 14 21
2 3 | 0

$(x-7)(2x+3)$

Example 3: Show that $(x-2)$ and $(x+3)$ are factors of $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$

*List all real zeros!

2 | 2 7 -4 -27 -18
↓ 4 22 36 18
-3 | 2 11 18 9 | 0
↓ -6 -15 -9
2 5 3 | 0

$(x-2)(x+3)(2x^2+5x+3)$
$(x-2)(x+3)(2x^2+2x+3x+3)$
$2x(x+1)$ $3(x+1)$
$(x-2)(x+3)(2x+3)(x+1)$
$x=2$ $x=-3$ $x=-\frac{3}{2}$ $x=-1$

the most amount of zeros is the highest degree!

Practice: Evaluate each of the following functions using the remainder theorem and synthetic division.

1) $f(x) = -x^3 + 6x - 7$ and $x - 2$

2) $f(x) = x^4 + 3x^3 - 17x^2 + 2x - 7$ and $x - 3$

3) $f(x) = x^5 - 47x^3 - 16x^2 + 8x + 52$ and $x - 7$

4) $f(x) = 6x^4 + 5x^2 - 8x + 3$ and $x + 8$

Directions: Determine if each of the following binomials are factors of the polynomial.

5) $f(x) = x^3 - x^2 - x - 2$ with a factor of $x - 2$

6) $f(x) = x^5 - 25x^3 - 7x^2 - 37x - 18$ with a factor of $x + 5$

7) $f(x) = x^4 - 8x^3 - x^2 + 62x - 34$ with a factor of $x - 7$

8) $f(x) = 8x^5 + 32x^4 + 5x + 20$ with a factor of $x + 4$

Directions: Using synthetic division to show that x is a solution to the polynomial equation and use the result to factor the polynomial completely. List all real zeros.

9) $f(x) = x^3 - 28x + 480$ and $x = -4$

10) $f(x) = 2x^3 - 15x^2 + 27x - 10$ and $x = \frac{1}{2}$

Directions: Verify the given factor of the function, find the remaining factors, use your results to write the complete factorization. List all real zeros.

11) $f(x) = 3x^3 + 2x^2 - 19x + 6$ with a factor of $(x + 3)$

12) $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$ with factors of $(x - 5) and\ (x + 4)$