

Factoring Perfect Cubes and Higher Powers

Kickoff: Solve each of the following:

1) $m^2 - 9m = 0$
 $m(m-9) = 0$
 $m = 0$ or $m - 9 = 0$
 $m = 9$

3) $r^2 - 3 = 22$
 $+3 +3$
 $\sqrt{r^2} = \sqrt{25}$
 $r = \pm 5$

2) $3b^2 + 6 = 11$
 $-6 -6$
 $\frac{3b^2}{3} = \frac{-5}{3}$
 $b = \pm \sqrt{\frac{5}{3}}$

4) $(x-4)^2 = 162$
 $\sqrt{(x-4)^2} = \sqrt{162}$
 $x-4 = \pm 9$
 $x-4 = 9$ $x-4 = -9$
 $+4 +4$ $+4 +4$
 $x = 13$ $x = -5$

Perfect Cubes

a product of multiplying the same # 3 times

Perfect Cubes	Cube Root
$1 \cdot 1 \cdot 1 = 1$	$\sqrt[3]{1} = 1$
$2 \cdot 2 \cdot 2 = 8$	$\sqrt[3]{8} = 2$
$4 \cdot 4 \cdot 4 = 64$	$\sqrt[3]{64} = 4$
$6 \cdot 6 \cdot 6 = 216$	$\sqrt[3]{216} = 6$
$7 \cdot 7 \cdot 7 = 343$	$\sqrt[3]{343} = 7$
$x \cdot x \cdot x = x^3$	$\sqrt[3]{x^3} = x$
$x^2 \cdot x^2 \cdot x^2 = x^6$	$\sqrt[3]{x^6} = x^2$
$x^3 \cdot x^3 \cdot x^3 = x^9$	$\sqrt[3]{x^9} = x^3$

1) Multiply: $(a-b)(a^2+ab+b^2)$
 $a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
 $a^3 - b^3$

Difference of perfect cubes.
 2) Multiply: $(a+b)(a^2-ab+b^2)$
 $a^3 - a^2b + ab^2 + ba^2 - ab^2 + b^3$
 $a^3 + b^3$

Sum of perfect cubes.

Sum/Difference of Perfect Cubes:

SOAP

a^3
 $3ab^2$
 $3a^2b$
 b^3
 S
 O
 A
 P

Factor: $(a+b)^3 = (a+b)(a^2 - ab + b^2)$

Factor: $(a-b)^3 = (a-b)(a^2 + ab + b^2)$

Ex1: $x^3 + 27$ $a = \sqrt[3]{x^3} = x$ $b = \sqrt[3]{27} = 3$
 $(a+b)(a^2 - ab + b^2)$
 $(x+3)(x^2 - 3x + 9)$

Ex2: $64x^3 - 8y^3$ $a = \sqrt[3]{64x^3} = 4x$ $b = \sqrt[3]{8y^3} = 2y$
 $(4x-2y)((4x)^2 + (4x)(2y) + (2y)^2)$
 $(4x-2y)(16x^2 + 8xy + 4y^2)$

Ex3: $250 + 2x^3$ $a = \sqrt[3]{250} = 5$ $b = \sqrt[3]{x^3} = x$
 $2(5+x)(5^2 - 5x + x^2)$
 $2(5+x)(25 - 5x + x^2)$

Ex4: $8x^4 - x$

Higher Powers

*NOTES: difference - master product.

Ex1: $x^6 - 2x^4 + 1$
 $x^6 - x^4 - x^4 + 1$
 $x^4(x^2 - 1) - 1(x^4 - 1)$
 $x^4(x-1)(x+1) - 1(x^2-1)(x^2+1)$
 $(x^2-1)(x^2+1)(x^2-1)(x^2+1)$

Ex3: $x^4 + 7x^2 + 6$

Ex2: $x^4 - 7x^2 + 12$

Ex4: $-2r^8 + 5r^4 - 2$
 $-1(2r^8 - 5r^4 + 2)$
 $-1(2r^8 - 4r^4 - r^4 + 2)$
 $-1(2r^4(r^4 - 2) - 1(r^4 - 2))$
 $-1(2r^4 - 1)(r^4 - 2)$

Practice:

1) $x^4 + 8x^2 - 2$

2) $x^3 - 64$

3) $-27d^3 + 125$

4) $-25p^4 + 160p^2 + 320$

5) $10a^3 + 17a^2 + 6a$

11) $x^5 + x^4 - 7x^3 - 7x^2 + 12x + 12$

6) $-9n^{10} + 58n^5 - 24$

12) $x^3 - 10x^2y + 24y^2x$

7) $8a^3 + 125$

13) $a^3 + 343b^3$

8) $x^3 - 216y^3$

14) $-a^3 - 8$

9) $2x^4 - 6x^2y^2 - 108y^4$

15) $250x^4 + 128x$

10) $-16x^4 + 58x^2 + 24$

16) $-2r^8 + 5r^4 - 2$