

Lesson 2.6- Objective: SWBAT determine the end behavior of a polynomial function.

Kickoff

Complete questions 2 & 6 in yesterdays packet!

$$2) a^2 + 3a + 2 < -3(a+2)$$

$$\begin{aligned} a^2 + 3a + 2 &< -3(a+2) \\ a^2 + 6a + 8 &< 0 \\ (a+4)(a+2) &< 0 \\ a=-4 & \quad a=-2 \end{aligned}$$

$$\frac{(-4, -2)}{3} \quad (-3)^2 + 3(-3) < 0$$

$$a^2 + 6a + 9 = 8 + 9$$

$$(a+3)^2 = 1$$

$$a+3 = \pm 1$$

$$\begin{cases} a+3=1 \\ a+3=-1 \end{cases}$$

$$\begin{cases} a=-2 \\ a=-4 \end{cases}$$

$$\begin{aligned} 5) 3x^2 + 7x \leq -2 \\ 3x^2 + 7x + 2 \leq 0 \\ (3x^2 + 6x)(x+2) \leq 0 \\ 3x(x+2)(x+1) \leq 0 \\ 3x=0 \quad x+2=0 \\ x=0 \quad x=-2 \\ \text{Graph: } \begin{array}{|c|c|c|c|c|c|c|} \hline & -3 & -2 & -1 & 0 & 1 & 2 \\ \hline f(x) & & & & \text{---} & & \\ \hline \end{array} \end{aligned}$$

$$\begin{aligned} 7) 2x^2 > 9x + 18 \\ 2x^2 - 9x - 18 > 0 \\ \frac{9 \pm \sqrt{81 + 144}}{4} \\ \frac{9 \pm 15}{4} \\ \frac{9+15}{4} = 6 \quad \frac{9-15}{4} = -\frac{6}{2} = -3 \end{aligned}$$

$$\begin{aligned} 11) \frac{x^2 + 4}{x^2 - 4} \geq 2x^2 - 3x \\ 5x^2 + 12x + 16 \geq x^2 - 3x - 4 \\ 29x^2 + 15x + 20 \geq (x-4)(x+1) \\ 29x^2 + 15x + 20 \geq x^2 + x - 4 \\ 28x^2 + 14x + 24 \geq 0 \\ 2(14x^2 + 7x + 12) \geq 0 \\ 14x^2 + 7x + 12 \geq 0 \\ (2x+3)(7x+4) \geq 0 \\ \text{Graph: } \begin{array}{|c|c|c|c|c|c|c|} \hline & -3 & -2 & -1 & 0 & 1 & 2 \\ \hline f(x) & & & & \text{---} & & \\ \hline \end{array} \end{aligned}$$

$$6) x^2 > 8$$

$$X = \pm \sqrt{8}$$

$$\begin{array}{|c|c|c|c|c|c|c|} \hline & -3 & -\sqrt{8} & 0 & \sqrt{8} & 3 \\ \hline f(x) & & & \text{---} & & & \\ \hline \end{array}$$

$$\begin{array}{ccc} -3^2 < 8 & 0^2 < 8 & 3^2 < 8 \\ \cancel{9 < 8} & \cancel{0 < 8} & \cancel{9 < 8} \end{array}$$

The degree of a polynomial is the highest exponent in the equation.

The leading coefficient of a polynomial is the coefficient in front of the variable that includes the degree of the polynomial.

Try this: Determine the degree and leading coefficient

$$1) f(x) = -x^3 + 4x^2 - 2x + 1$$

$$LC: -1$$

$$D: 3$$

$$2) f(x) = (2x-3)^2$$

$$(2x-3)(2x-3)$$

$$4x^2$$

$$LC: 4$$

$$D: 2$$

$$3) f(x) = -(2x+4)^2$$

$$-4x^2$$

$$LC: -4$$

$$D: 3$$

End Behavior of a Polynomial- $f(x) = ax^n$

Think about the basic quadratic and cubic functions when completing the end behavior!

-When "n" is odd and $a > 0$ then:

$$\begin{array}{ccc} + & \nearrow & X \rightarrow \infty f(x) \rightarrow \infty \\ & \searrow & X \rightarrow -\infty f(x) \rightarrow -\infty \end{array}$$

-When "n" is odd and $a < 0$ then:

$$\begin{array}{ccc} - & \nearrow & X \rightarrow \infty f(x) \rightarrow -\infty \\ & \searrow & X \rightarrow -\infty f(x) \rightarrow \infty \end{array}$$

-When "n" is even and $a > 0$ then:

$$\begin{array}{ccc} + & \nearrow & X \rightarrow \infty f(x) \rightarrow \infty \\ & \searrow & X \rightarrow -\infty f(x) \rightarrow \infty \end{array}$$

-When "n" is even and $a < 0$ then:

$$\begin{array}{ccc} - & \nearrow & X \rightarrow \infty f(x) \rightarrow -\infty \\ & \searrow & X \rightarrow -\infty f(x) \rightarrow -\infty \end{array}$$

Quiz Friday

- ① Forms of a quadratic
- ② End Behavior
- ③ Multiplicity
- ④ Sketching a polynomial
*without a calculator!
- ⑤ Quadratic Inequalities

To determine end behavior, use the leading coefficient test!

Leading Coefficient Test:

Degree

If the degree is even, the ends will be:

- Always positive
- Always negative

If the degree is odd the ends be:

- both positive + negative
- (1, 3)
- (2, 4)

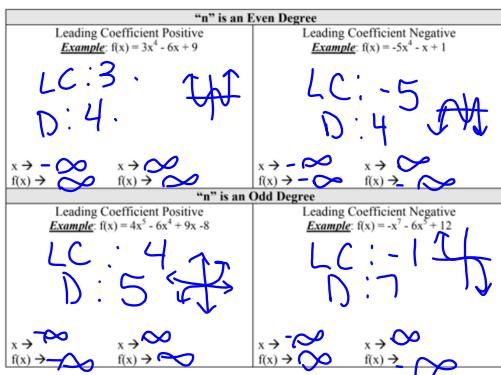
Coefficient:

If the leading coefficient is:

- negative $\rightarrow \curvearrowleft$
- positive $\rightarrow \curvearrowright$

If the leading coefficient is negative

- negative (2, 4)
- positive (1, 3)

End Behavior:Determine the following as x decreases then $f(x)$ increases or decreases
x increases then $f(x)$ increases or decreases

1) $f(x) = x^2 + 4x - 5$

Degree 2 Even or Odd
Leading Coefficient 1

End Behavior
 $x \rightarrow \infty f(x) \rightarrow \infty$
 $x \rightarrow -\infty f(x) \rightarrow \infty$

2) $f(x) = 8x^2 - 2x^4$

Degree 4 Even or Odd
Leading Coefficient -2

End Behavior
 $x \rightarrow \infty f(x) \rightarrow -\infty$
 $x \rightarrow \infty f(x) \rightarrow \infty$

3) $f(x) = x^2(x - 4)$

Degree _____ Even or Odd
Leading Coefficient _____

End Behavior

4) $f(x) = -x(x - 2)^2(x + 2)^2$

Degree _____ Even or Odd
Leading Coefficient _____

End Behavior

5) $f(x) = -x(x + 1)(x - 2)^4(x - 3)$

Degree _____ Even or Odd
Leading Coefficient _____

End Behavior

6) $f(x) = -x^3 - 10x^2 + 25x$

Degree _____ Even or Odd
Leading Coefficient _____

End Behavior