Lesson 2.7- Objective: SWBAT determine the multiplicity of a polynomial function and sketch it by hand.

Kickoff

Determine the end behavior of the following functions:

$(3x)^2 = 9x^2(-x) = -9x^3$ neg.

1) $f(x) = (3x-2)^2(-x)$

① LC: $-9$
D: 3
$x \to \infty$ $f(x) \to -\infty$
$x \to -\infty$ $f(x) \to \infty$

2) $f(x) = -4x^2 + 9x^6$

② LC: 9
D: 6
$x \to \infty$ $f(x) \to \infty$
$x \to -\infty$ $f(x) \to \infty$

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$x^3$

3) $f(x) = x^2(x-4)$

Degree 3 Even or Odd

Leading Coefficient 1

End Behavior

$x \to -\infty$ $f(x) \to -\infty$

$x \to \infty$ $f(x) \to \infty$

$-x \cdot x^2 \cdot x^2 = -x^5$

4) $f(x) = -x(x-2)^2(x+2)^2$

Degree 5 Even or Odd

Leading Coefficient $-1$

End Behavior

$x \to -\infty$ $f(x) \to \infty$

$x \to \infty$ $f(x) \to -\infty$

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$-x \cdot x \cdot x^4 \cdot x = -x^7$

5) $f(x) = -x(x+1)(x-2)^4(x-3)$

Degree 7 Even or Odd

Leading Coefficient $-1$

End Behavior

$x \to -\infty$ $f(x) \to \infty$

$x \to \infty$ $f(x) \to -\infty$

6) $f(x) = -x^3 - 10x^2 + 25x$

Degree 3 Even or Odd

Leading Coefficient $-1$

End Behavior

$x \to -\infty$ $f(x) \to \infty$

$x \to \infty$ $f(x) \to -\infty$

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7) $f(x) = 2x^2 - 22x + 60$

LC = 2 D = 2 Even

$x \to -\infty$ $f(x) \to \infty$

$x \to \infty$ $f(x) \to \infty$

8) $d(x) = -3x^3 - 23x^2 + 36x$

LC = $-3$ D = 3 odd

$x \to -\infty$ $f(x) \to \infty$

$x \to \infty$ $f(x) \to -\infty$

9) $f(x) = -6x^2 - 9x$

LC = $-6$ D: 2 even

$x \to \infty$ $f(x) \to -\infty$

$x \to -\infty$ $f(x) \to -\infty$

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8) $f(x) = 8x^3 - 27$

LC: 8 D: 3 odd

$x \to -\infty$ $f(x) \to -\infty$

$x \to \infty$ $f(x) \to \infty$

10) $f(x) = -125x^3 + 216$

LC = $-125$ D: 3

$x \to -\infty$ $f(x) \to \infty$

$x \to \infty$ $f(x) \to -\infty$

11) $f(x) = -4x(5x-3)(2x+5)^3(x-1)$

$-4x \cdot 5x \cdot 8x^3 \cdot x = -160x^6$

$x \to -\infty$ $f(x) \to -\infty$

$x \to \infty$ $f(x) \to -\infty$

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12) $f(x) = 5(x-2)^2(x+2)(x-2)$

$5x^2 \cdot x \cdot x = 5x^4$

$x \to -\infty$ $f(x) \to \infty$

$x \to \infty$ $f(x) \to \infty$

## Multiplicity and the Intermediate Value Theorem

Try this: Find the zero's and draw a sketch of the graph of the following:

1) $f(x) = (x+2)^{\boxed{2}}$

$\sqrt{(x+2)^2} = \sqrt{0}$

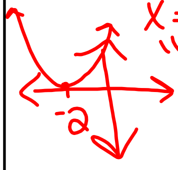
$x + 2 = 0$

$-2 \quad -2$

$x = -2$

"bounce"

tangent



2) $f(x) = (x+2)^1(x-2)^1 = 0$

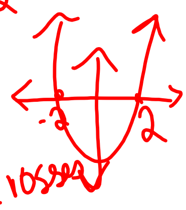
$x^2$

LC: 1

D: 2

$x + 2 = 0 \quad x - 2 = 0$

$x = -2 \quad x = 2$



Crosses

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**Multiplicity (must be in factored form)**

Must have the roots

*If the exponent of a factor is even, the graph is tangent to the x-axis, it will be tangent at its zero's (bounce)

*If the exponent of a factor is odd, the graph will cross the x-axis, it will cross at its zero's

Factor, find the zero's and it's multiplicity.

1) $f(x) = -2x^4 + 2x^2$

$-2x^2(x^2 - 1)$

bounce, cross, cross

$-2x^2(x-1)(x+1)$

$-2x^2 = 0 \quad x - 1 = 0 \quad x + 1 = 0$

$\sqrt{x^2} = \sqrt{0} \quad x = 1 \quad x = -1$

$x = 0$

Bounce | cross | cross

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**To Sketch a Polynomial Function**

1. Use the leading coefficient test to find the end behavior
2. Find the multiplicity
3. Sketch the graph as a continuous curve

$f(x) = -2x^4 + 2x^2$

LC: $-2$

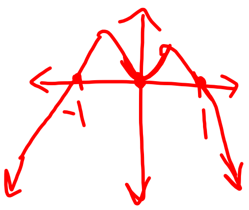
D: 4

$x \to -\infty \quad f(x) \to -\infty$

$x \to \infty \quad f(x) \to -\infty$

$x = 0$ bounce

$x = 1 \quad x = -1$ cross



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2) $f(x) = x^3 - x^2 - 2x$

D: 3

LC = 1

$x(x^2 - x - 2)$

$x^1(x+1)^1(x-2)^1$

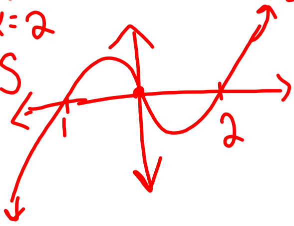
$x \to \infty \quad f(x) \to \infty$

$x \to -\infty \quad f(x) \to -\infty$

$x = 0 \quad x + 1 = 0 \quad x - 2 = 0$

$x = -1 \quad x = 2$

ALL CROSS



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3) $f(x) = 8x^2 - 2x^4$ LC $= -2$ D: 4

As $x \to -\infty$, $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to -\infty$

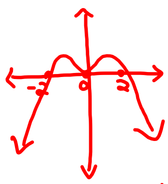
$8x^2 - 2x^4$

$2x^2(4 - x^2)$

$2x^2 = 0 \quad 4 - x^2 = 0$

$\sqrt{x^2} = \sqrt{0} \quad \sqrt{4} = \sqrt{x^2}$

$x = 0$ bounce $\quad x = \pm 2$ cross



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**The Intermediate Value Theorem**

The x interval in which you would find a zero.

when you can't factor!

Example: in what interval of one unit will you find a zero

| x | f(x) |
|---|---|
| -3 | -4 |
| -2 | -2 |
| -1 | 5 |
| 0 | 7 |
| 1 | -3 |
| 2 | -5 |

determine where the signs change

the zeros are between

$x = -2$ and $x = -1$

$x = 0$ and $x = 1$

Example: Sketch a graph of $f(x) = -x^2 + 7x - 5$ using the Intermediate Value Theorem.

| $x$ | $f(x)$ |
|---|---|
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |

1) $f(x) = x^2(x-2)$

2) $f(x) = -(x+1)(x-2)^4(x-3)$

3) $f(x) = -x^3 + 9x$

4) $g(x) = x^4 - 10x^2 + 9$