

Lesson 25- Remainder and Factor Theorem.notebook

Lesson 2.11- Objective: SWBAT apply the remainder and factor theorem with polynomials.

Take out your homework and answer the following:
 1) Divide $2x^2 - 4x + 7$ by $x + 1$ using synthetic division.
 2) Graph the following polynomial WITHOUT a calculator. $f(x) = x^3 - 4x^2 + x + 4$ (Be sure to state ALL necessary parts)

①
$$\begin{array}{r|rrrr} 1 & 2 & -4 & 7 & \\ & \downarrow & 2 & -2 & \\ \hline & 2 & -2 & 5 & \end{array}$$

 $2x - 2 + \frac{5}{x+1}$

② $f(x) = (x^3 - 4x^2)(x + 4)$
 LC: 1 $\rightarrow x^3(x-4) - 1(x-4)$
 D: 3 $\rightarrow (x^2-1)(x-4)$
 $(x+1)(x-1)(x-4)$
 $x \rightarrow \infty f(x) \rightarrow \infty x+1=0 \quad x-1=0 \quad x-4=0$
 $x \rightarrow \infty f(x) \rightarrow \infty \quad x=-1 \quad x=1 \quad x=4$
 (Cross) Cross Cross

*total mins/max is one less than your degree

③ $x + 4$
 ④ $x^2 - x - 3$
 ⑤ $7x^3 - 3x^2 + 3$
 ⑥ $x^2 - \frac{4}{x-2}$
 ⑦ $2x^2 - 3x - 3 + \frac{2}{x+1}$
 ⑧ $6x^4 - 3x^3 - 2x^2 + 1 - \frac{10}{x+4}$
 ⑨ $2x^2 + 4x + 3 - \frac{1}{x-2}$
 ⑩ $4x^2 - 32x + 241 - \frac{1911}{x+8}$

Remainder and Factor Theorem
 Remainder Theorem- If you divide a polynomial by $(x - k)$ then the remainder is $f(k)$.
 Example 1: $f(x) = 3x^3 + 8x^2 + 5x - 7$ find $f(-2)$ using synthetic division.

$$\begin{array}{r|rrrr} -2 & 3 & 8 & 5 & -7 \\ & \downarrow & -6 & -4 & -2 \\ \hline & 3 & 2 & 1 & -9 \end{array}$$

$f(-2) = 3(-2)^3 + 8(-2)^2 + 5(-2) - 7 = -9$

Factor Theorem- A polynomial $f(x)$ has a factor of $(x - k)$ if $f(k) = 0$ (there is no remainder!).
 Example 2: Show that $(x - 7)$ is a factor of $2x^2 - 11x - 21$

~~$$\begin{array}{r|rrrr} 7 & 2 & 0 & -11 & -21 \\ & \downarrow & 14 & & \\ \hline & 2 & 14 & & \end{array}$$~~

$$\begin{array}{r|rrrr} 7 & 2 & -11 & -21 \\ & \downarrow & 14 & 7 \\ \hline & 2 & 3 & 0 \end{array}$$

① $(x-7)(2x+3)$

Example 3: Show that $(x - 2)$ and $(x + 3)$ are factors of $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$

*List all real zeros!

$$\begin{array}{r|rrrrr} 2 & 2 & 7 & -4 & -27 & -18 \\ & \downarrow & 4 & 22 & 36 & 18 \\ \hline & 2 & 11 & 18 & 9 & 0 \\ -3 & \downarrow & -6 & -15 & -9 & 0 \\ \hline & 2 & 5 & 3 & 0 & \end{array}$$

$(x-2)(x+3)(2x^2+5x+3)$
 $(x-2)(x+3)(2x^2+2x+3x+3)$
 $\downarrow \quad \quad \quad 2x(x+1) \quad 3(x+1)$
 $(x-2)(x+3)(2x+3)(x+1)$
 $x=2 \quad x=-3 \quad x=-\frac{3}{2} \quad x=-1$

the most amount of zeros is the highest degree!

Practice: Evaluate each of the following functions using the remainder theorem and synthetic division.

1) $f(x) = -x^3 + 6x - 7$ and $x - 2$

2) $f(x) = x^4 + 3x^2 - 17x^2 + 2x - 7$ and $x - 3$

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3) $f(x) = x^5 - 47x^3 - 16x^2 + 8x + 52$ and $x - 7$

4) $f(x) = 6x^4 + 5x^2 - 8x + 3$ and $x + 8$

Directions: Determine if each of the following binomials are factors of the polynomial.

5) $f(x) = x^3 - x^2 - x - 2$ with a factor of $x - 2$

6) $f(x) = x^5 - 25x^3 - 7x^2 - 37x - 18$ with a factor of $x + 5$

7) $f(x) = x^4 - 8x^3 - x^2 + 62x - 34$ with a factor of $x - 7$

8) $f(x) = 8x^5 + 32x^4 + 5x + 20$ with a factor of $x + 4$

Directions: Using synthetic division to show that x is a solution to the polynomial equation and use the result to factor the polynomial completely. List all real zeros.

9) $f(x) = x^3 - 28x + 480$ and $x = -4$

10) $f(x) = 2x^3 - 15x^2 + 27x - 10$ and $x = \frac{1}{2}$

Directions: Verify the given factor of the function, find the remaining factors, use your results to write the complete factorization. List all real zeros.

11) $f(x) = 3x^3 + 2x^2 - 19x + 6$ with a factor of $(x + 3)$

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12) $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$ with factors of $(x - 5)$ and $(x + 4)$