Objective: SWBAT use the rational zero test and Descartes rule of signs to determine possible zeros.

Kickoff
Take out your homework and weekly homework quiz.
Answer the following:
1) Given the polynomial and one zero of the polynomial. Find the remaining factors and write the factorization. Then find all the real zeros.
a) $f(x) = x^3-6x^2+11x-6$ and $x = 1$
b) $f(x) = 2x^3-x^2-10x+5$ and $x=1/2$

a) 1 | 1 -6 11 -6
↓ 1 -5 6
1 -5 6 | 0

$x^2-5x+6$
$(x^2-2x)(-3x+6)$
$x(x-2)-3(x-2)$
$(x-2)(x-3)=0$
$\boxed{x=1 \quad x=2 \quad x=3}$

b) $\frac{1}{2}$ | 2 -1 -10 5
↓ 1 0 -5
2 0 -10 | 0

$2x^2+\cancel{0x}-10$
$2x^2-10=0$
$+10 \quad +10$
$\frac{2x^2}{2}=\frac{10}{2}$
$\sqrt{x^2}=\sqrt{5}$
$x=\frac{1}{2} \quad x=\pm\sqrt{5}$

1) -3
2) 24963
3) NO
4) yes
5) $x=1/2$, $x=5$, $x=2$
6) $x=2$, $x=-3$, $x=-3/2$, $x=-1$

3) 7 | 1 -8 -1 62 -34
↓ 7 -7 -56 42
1 -1 -8 6 | 8
NO

4) -4 | 8 32 0 0 5 20
↓ -32 0 0 0 -20
8 0 0 0 5 | 0
yes

5) $\frac{1}{2}$ | 2 -15 27 -10
↓ 1 -7 10
2 -14 20 | 0

$40x^2$ $2x^2-14x+20$
$2x^2-10x-4x+20$
$2x(x-5)-4(x-5)$
$(2x-4)(x-5)=0$
$x=\frac{1}{2} \quad x=\frac{4}{2}=2 \quad x=5$

Rational Zeros Test & Descartes Rule

Try This: Given the following functions evaluate:

$f(x)=x^2$ $g(x)=x^3$ $h(x)=x^4$ $k(x)=x^5$ $n(x)=x^6$

$f(-x)=(-x)^2=x^2$ $g(-x)=(-x)^3=-x^3$ $h(-x)=(-x)^4=x^4$ $k(-x)=(-x)^5=-x^5$ $n(-x)=(-x)^6=x^6$

*even exponent is a positive answer

**Rational Zero Test**
Possible rational zeros- relates the leading coefficient and the constant term of a polynomial

$$Possible\ Rational\ Zeros = \frac{factors\ of\ the\ consant\ term}{factors\ of\ leading\ coefficient}$$

**When the leading coefficient is 1, possible rational zeros are the factors of the constant term.**

Examples:
1) $f(x) = x^3-5x+2$
Possible Rational Zeros= $\frac{\pm 1, \pm 2}{\pm 1}$

$\frac{1}{1}, -\frac{1}{1}, \frac{2}{1}, -\frac{2}{1}$
$1, -1, 2, -2$
possible zeros

Test the Zeros:

$f(1)=(1)^3-5(1)+2=-2$
$f(-1)=(-1)^3-5(-1)+2=6$
$\boxed{f(2)=(+2)^3-5(2)+2=0}$
$f(-2)=(-2)^3-5(-2)+2=4$

$x=2$ rational zero

2) $f(x)=1x^2-6x+1$
Possible Rational Zeros= $\frac{\pm 1}{\pm 1}$

1, -1

Test the Zeros:

$f(1)=(1)^2-6(1)+1=-4$
$f(-1)=(-1)^2-6(-1)+1=8$
no rational zeros

3) $f(x)=2x^3+3x^2-8x+3$
Possible Rational Zeros= $\frac{\pm 1, \pm 3}{\pm 1, \pm 2}$

$\frac{1}{1}, \frac{-1}{1}, \frac{1}{2}, \frac{-1}{2}$
$\frac{3}{1}, \frac{-3}{1}, \frac{3}{2}, \frac{-3}{2}$
8 possible zeros

Test the Zeros:

$f(1)=2(1)^3+3(1)^2-8(1)+3=0$
$x=1$

1 | 2 3 -8 3
↓ 2 5 -3
2 5 -3 | 0

$2x^2+5x-3$
$2x^2+6x-x-3$
$2x(x+3)-1(x+3)$
$(2x-1)(x+3)=0$
$2x-1=0 \quad x+3=0$
$x=1 \quad x=1/2 \quad x=-3$

4) $f(x) = 8x^3 - 12x^2 - 4x + 10$

Possible Rational Zeros=

$\frac{\pm 10, \pm 5, \pm 2, \pm 1}{\pm 1, \pm 8, \pm 2, \pm 4}$

Test the Zeros:

NO Rational zeros

$\cancel{\frac{10}{1}}, \cancel{-\frac{10}{1}}, \cancel{\frac{10}{8} = \frac{5}{4}}, \cancel{-\frac{10}{8} = -\frac{5}{4}}, \cancel{\frac{10}{2} = 5}, \cancel{-\frac{10}{2} = -5},$

$\cancel{\frac{10}{4} = \frac{5}{2}}, \cancel{-\frac{10}{4} = -\frac{5}{2}}, \cancel{\frac{5}{8}}, \boxed{-\frac{5}{8}}, \cancel{\frac{2}{1}}, \cancel{-\frac{2}{1}}, \boxed{\cancel{\frac{2}{8} = \frac{1}{4}}},$

$\boxed{-\frac{2}{8} = -\frac{1}{4}}, \cancel{\frac{2}{2} = 1}, \cancel{-\frac{2}{2} = -1}, \cancel{\frac{2}{4} = \frac{1}{2}}, \boxed{-\frac{2}{4} = -\frac{1}{2}}, \cancel{\frac{1}{8}}, \boxed{-\frac{1}{8}}$

20 possibilities!

56) $x = 4 \quad x = 2/3 \quad x = 2$

58) $x = -1 \quad x = \frac{5}{2} \quad x = \frac{3}{2}$

**Descartes Rules of Signs**

This will tell us what kind of signs the zeros will have.

* Look at $f(x)$ and identify how many sign changes there are, this will give your at most positive real zeros.

* Look at $f(-x)$, so substitute in and simplify then identify how many sign changes there are. This will give your at most positive negative real zeros.

Examples: * Describe possible * more than 1 zero, count down 2.

State the possible number of positive and negative zeros for each function.

1) $f(x) = 5x^4 + 14x^2 - 32$

1 possible positive real zero + negative

$f(-x) = 5(-x)^4 + 14(-x)^2 - 32$

$5x^4 + 14x^2 - 32$

2) $f(x) = 3x^4 - 35x^2 + 12$

2 possible positive and neg or 0 possible positive and neg.

$f(-x) = 3(-x)^4 - 35(-x)^2 + 12$

$= 3x^4 - 35x^2 + 12$

3) $f(x) = 8x^6 - 2x^5 + 22x^4 - 10x^3 + 12x^2 - 6x$

5 or 3 or 1 possible positive real zeros

$f(-x) = 8(-x)^6 - 2(-x)^5 + 22(-x)^4 - 10(-x)^3 + 12(-x)^2 - 6(-x)$

$f(-x) = 8x^6 + 2x^5 + 22x^4 + 10x^3 + 12x^2 + 6x$

0 possible negative real zeros.

HW pg 171 #~~56-62~~ even 56 + 58

**ignore the directions and do the following

a) Descartes Rule

b) find all possible real zeros with the rational zero test

c) test and find the real zeros