

Lesson 35- Objective: SWBAT use the fundamental theorem of algebra to find the zeros of a polynomial.

Kickoff: Find the zeros of the polynomial.  
 $f(x) = 4x^4 + 5x^3 - 4x^2 - 5$

$\pm 1, \pm 5 = \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{4}, \pm \frac{5}{4}$   
 $\pm 1, \pm 2, \pm 4$

$f(-1) = 0$   
 $f(1) = 0$

4	0	5	0	-4	0	5
	↓	4	9	9	5	5
-1	4	4	9	9	5	5
	↓	4	0	-9	0	5
		4	0	0	5	0

$f(x) = (x-1)(x+1)(4x^2+9x+5)$   
 $x-1=0 \quad x+1=0 \quad 4x^2+9x+5=0$   
 $x=1 \quad x=-1$

$4x^2+9x+5=0$   
 $4x^2(x+1)+5(x+1)$   
 $(4x^2+5)(x+1)$   
 $\frac{4x^2+5=0}{x^2+\frac{5}{4}} \quad \frac{x^2+1=0}{x^2=-1}$   
 $\sqrt{x^2+\frac{5}{4}} \quad \sqrt{x^2+1}$   
 $x = \pm \frac{\sqrt{5}}{2}$   
 $x = \pm i$

Zeros  $\{1, 1, \frac{\sqrt{5}}{2}, -\frac{\sqrt{5}}{2}, -i, i\}$

(22)  $\{1, i, i, i\}$

(24)  $\{-1, -5, 2i, -5+2i\}$

(29)  $\{2, 2, 2i, -2i\}$

(31)  $\{-2, \frac{1}{2}, i, -i\}$

-1	1	11	39	29	
	↓	-1	-10	-29	
		1	10	29	0

$x^2 + 10x + 29 = 0$   
 $x^2 + 10x + 25 = -4$   
 $(x+5)^2 = -4$

(31)  $2x^2 + 2 = 0$   
 $\frac{2x^2}{2} = \frac{-2}{2}$   
 $\sqrt{x^2} = \sqrt{-1}$   
 $x = \pm i$

Fundamental Theorem of Algebra

Try This: Find the zeros.

$f(x) = x^2 + 6x + 45 = 0$

Zeros  $\{ -3+6i, -3-6i \}$

$x^2 + 6x + 9 = -45 + 9$   
 $\sqrt{(x+3)^2} = \sqrt{-36}$   
 $x+3 = \pm 6i$   
 $x = -3 \pm 6i$

Linear Factors  $(x+3-6i)(x+3+6i)$

Solving Polynomial Functions

- Factoring
- Higher Power Factoring
- Use synthetic division to find factors

\* Only use Quadratic Formula  
 • And complete the square FOR QUADRATICS!  
 • Write as linear factors

Examples: Find the zeros and write as linear factors.

1)  $f(x) = x^3 + 4x$   
 $x(x^2+4) = 0$   
 $x = 0 \quad x^2+4 = 0$   
 $x^2 = -4$   
 $\sqrt{x^2} = \sqrt{-4}$   
 $x = \pm 2i$   
 Zeros  $\{0, 2i, -2i\}$   
 Linear factors  $(x)(x-2i)(x+2i)$

2)  $f(x) = x^4 - x^2 - 20$   
 $x^4 + 4x^2 - 5x^2 - 20$   
 $x^2(x^2+4) - 5(x^2+4)$   
 $(x^2+4)(x^2-5) = 0$   
 $x^2+4 = 0 \quad x^2-5 = 0$   
 $x = \pm 2i \quad x = \pm \sqrt{5}$   
 Zeros  $\{2i, 2i, \sqrt{5}, -\sqrt{5}\}$   
 LF  $(x+2i)(x-2i)(x+\sqrt{5})(x-\sqrt{5})$

3)  $f(x) = 5x^4 - 245 = 0$

$\frac{5x^4 - 245}{5} = 0$   
 $x^4 - 49 = 0$   
 $(x^2-7)(x^2+7) = 0$   
 $x^2-7 = 0 \quad x^2+7 = 0$   
 $\sqrt{x^2} = \sqrt{7} \quad \sqrt{x^2} = \sqrt{-7}$   
 $x = \pm \sqrt{7} \quad x = \pm i\sqrt{7}$   
 Zeros  $\{ \sqrt{7}, -\sqrt{7}, i\sqrt{7}, -i\sqrt{7} \}$   
 LF  $(x-\sqrt{7})(x+\sqrt{7})(x-i\sqrt{7})(x+i\sqrt{7})$

4)  $f(x) = x^3 + x^2 + 2x - 8$

Possible Zeros  
 $\pm 1, \pm 2, \pm 4, \pm 8$

$f(-2) = 0$   
 $f(1) = 0$

1	1	0	2	-8
	↓	1	2	-8
-2		1	2	-8
	↓	-2	2	-8
		1	4	-4
		↓	1	4
			1	4

$f(x) = (x-1)(x+2)(x^2 - x^2 + 4x - 4)$

$x-1=0 \Rightarrow x=1$   
 $x+2=0 \Rightarrow x=-2$

$(x^2 - x^2 + 4x - 4) = 0$   
 $x^2 - 4 = 0$   
 $(x+2)(x-2) = 0$   
 $x = \pm 2i$

Zeros:  $-2, 1, 1, 2i, -2i$   
 LF:  $(x+2)(x-1)(x-1)(x-2i)(x+2i)$

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Practice:

- 1)  $f(x) = x^4 - 16$
- 2)  $h(x) = x^4 - x^3 - 9x^2 - 3x - 36$
- 3)  $g(x) = x^4 - 4x^3 + 8x^2 - 8x$
- 4)  $j(x) = x^4 - 6x^3 - 11x^2 + 86x - 120$
- 5)  $f(x) = x^5 - 8x^4 + 87x^3 - 284x^2 + 268x$