Lesson 37- Objective: SWBAT determine the asymptotes and holes in a rational function.

Kickoff

Graph the polynomial function WITHOUT a calculator- $f(x)= x^3 - 13x^2 + 23x - 11$

LC: 1 $x \to \infty$ $f(x) \to \infty$
D: 3 $x \to -\infty$ $f(x) \to -\infty$

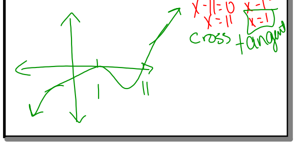
$\frac{\pm 11, \pm 1}{\pm 1} \to$ Possible Zeros $\pm 11, \pm 1$

$f(1) = (1)^3 - 13(1)^2 + 23(1) - 11 = 0$

1	1	-13	23	-11
	↓	1	-12	11
	1	-12	11	$\boxed{0}$

$f(x) = (x-1)(x^2 - 12x + 11)$

$x - 1 = 0$ $\quad$ $x^2 - 12x + 11 = 0$
$\boxed{x = 1}$ $\quad$ $(x-11)(x-1) = 0$
$x - 11 = 0$ $\quad$ $x - 1 = 0$
$x = 11$ $\quad$ $\boxed{x = 1}$
cross $\quad$ tangent



Rational Functions and Asymptotes

Try This: Find the domain

1) $f(x) = \frac{1}{x-1}$

$x - 1 = 0$

$x = 1$

$\mathbb{R}$ except 1

2) $f(x) = \frac{4x}{x^2-81}$

$x^2 - 81 = 0$

$\sqrt{x^2} = \sqrt{81}$

$x = \pm 9$

$\mathbb{R}$ except ± 9

3) $f(x) = \frac{5}{3x-12}$

$3x - 12 = 0$

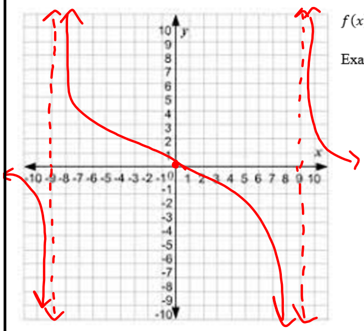
$3x = 12$

$x = 4$

$\mathbb{R}$ except 4

Asymptotes- a line that function approaches but never reaches as the function approaches it.

Vertical Asymptotes- where the domain is undefined.



$f(x) = \frac{N(x)}{D(x)}$, where $D(x) = 0$

Example: $f(x) = \frac{4x}{x^2-81}$

$x^2 - 81 = 0$

$\sqrt{x^2} = \sqrt{81}$

$x = \pm 9$

Horizontal Asymptotes

- $f(x)$ can have at most one horizontal asymptote depending on the degree of $N(x)$ and $D(x)$

Rules $\quad f(x) = \frac{ax^n}{bx^m}$

- If n < m (the power in the numerator is less than the power in the denominator), the horizontal asymptote is, $y = 0$ (x axis)
- If n = m, (the power in the numerator is equal to the power in the denominator), the horizontal asymptote is $y = a/b$ (the coefficients)
- If n > m (the power in the numerator is greater than the power in the denominator), there is none!

Ex: 1) $f(x) = \frac{x+4}{x^2-9}$

$\frac{x}{x^2}$ $\quad$ n < m $\quad$ y = 0

2) $f(x) = \frac{x-5}{x+8}$

$\frac{1x}{1x}$ $\quad$ n = m $\quad$ $y = \frac{1}{1} = 1$

3) $f(x) = \frac{x^2-6}{x+2}$

$\frac{x^2}{x}$ $\quad$ none

Holes- a point at which a graph is not connected but, can be made connected by filling in a single point.

To find a whole, we need to first simplify a rational function. If anything can be canceled, that is where a hole appears.

factor!

Ex: $f(x) = \frac{x+2}{x^2-x-6}$

$f(x) = \frac{\cancel{x+2}}{(x-3)\cancel{(x+2)}} = \frac{1}{x-3}$

$x + 2 = 0$

$x = -2$

$\frac{1}{-2-3} = \frac{-1}{5}$

$(-2, -1/5)$

****In order to find vertical asymptotes and holes, you fist must SIMPLIFY the rational function and then find them. *****

Ex: Find the vertical asymptotes, horizontal asymptotes and any holes, if any.

$f(x) = \frac{x^3-4x}{3x^3-6x^2-24x} = \frac{x(x^2-4)}{3x(x^2-2x-8)} = \frac{\cancel{x}\cancel{(x+2)}(x-2)}{3\cancel{x}(x-4)\cancel{(x+2)}}$

$\frac{x-2}{3(x-4)}$

VA $\to$ $\frac{3(x-4)}{3} = \frac{0}{3}$

$x - 4 = 0$

$x = 4$

HA $\to$ $\frac{1x^3}{3x^3}$

$y = 1/3$

holes $\quad x = 0$

$\frac{0-2}{3(0-4)} = \frac{-2}{-12} = \frac{1}{6}$

$(0, 1/6)$

$x + 2 = 0$
$-2 \quad -2$
$x = -2$

$\frac{-2-2}{3(-2-4)} = \frac{+4}{+18}$

$(-2, \frac{2}{9})$

Examples:

Find the vertical and horizontal asymptotes of the following functions.

1) $f(x) = \frac{3x}{x^2-4}$ $\frac{3x}{(x-2)(x+2)}$

VA: $x^2 - 4 = 0$

$+4 \quad +4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

holes

none

HA $\rightarrow \frac{3x^1}{x^2}$ $y = 0$

2) $f(x) = \frac{x+4}{3x-9}$

3) $f(x) = \frac{x-2}{x^2-4}$

4) $f(x) = \frac{4}{x^2+1}$

5) $f(x) = \frac{x^3+2x-8x}{-2x^2-4x+6}$

6) $f(x) = \frac{-x-4}{x^2-16}$