

Name: Answer key

Date: _____

Graphs of Rational Functions

Directions: Graph each rational function below by hand and then determine the values below. If the value does not exist, write "none".

1) $f(x) = \frac{x^2 + 6x + 8}{x^2 + 5x + 4}$

$\frac{(x+4)(x+2)}{(x+4)(x+1)} = \frac{x+2}{x+1}$

$\frac{x+2}{x+1} = 0$
 $x+2=0$
 $x=-2$

$\frac{0+2}{0+1} = \frac{2}{1}$

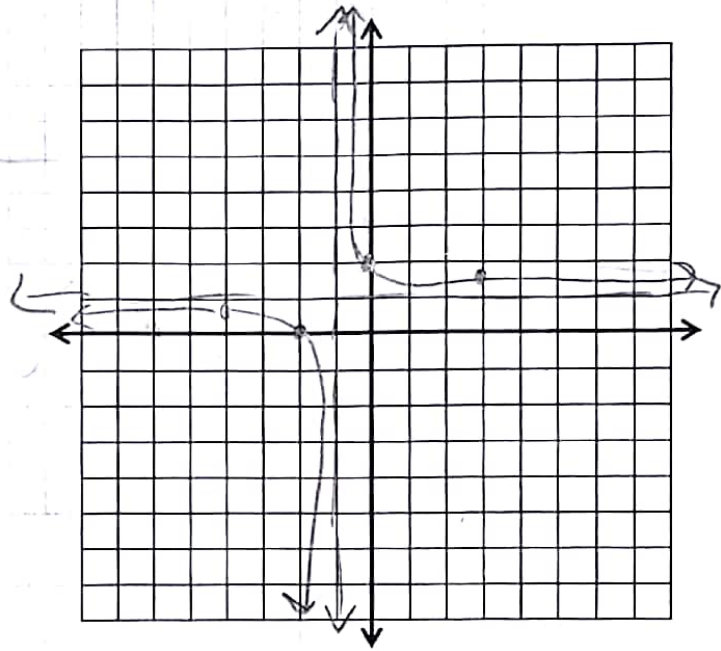
VA \rightarrow $x+4=0$ $x+1=0$
 $x=-4$ $x=-1$

H/A \rightarrow $\frac{x^2}{x^2} = y=1$

holes $(-4,$

$\frac{-4+2}{-4+1} = \frac{-2}{-3} = \frac{2}{3}$

$f(3) = \frac{3+2}{3+1} = \frac{5}{4} = \frac{3}{2}$



Domain:

$(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$

Vertical Asymptote(s):

$x = -1$

Hole(s):

$(-4, \frac{2}{3})$

Horizontal Asymptote(s):

$y = 1$

x-intercept(s):

$(-2, 0)$

y-intercept(s):

$(0, 2)$

Range (use graph):

$(-\infty, 1) \cup (1, \infty)$

$$2) f(x) = \frac{x-1}{x^2-1} \quad \frac{\cancel{x-1}}{(x+1)(\cancel{x-1})} = \frac{1}{x+1}$$

$$\frac{1}{0+1} = 1$$

$$\frac{1}{x+1} = 0$$

$$1 \neq 0$$

VA \rightarrow $x+1=0$
 $x=-1$

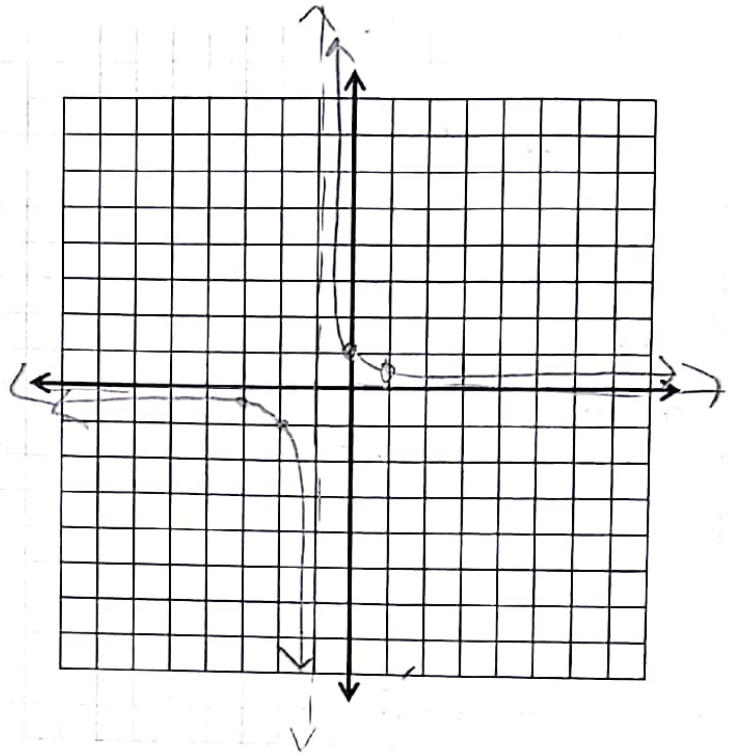
holes $x-1=0$
 $x=1$

$$\frac{1}{1+1} = \frac{1}{2}$$

H/A \rightarrow $\frac{x}{x^2} \rightarrow y=0$

$$f(-2) = \frac{1}{-2+1} = -1$$

$$f(-4) = \frac{1}{-4+1} = -\frac{1}{3}$$



Domain:

$$(-\infty, -1) \cup (1, \infty)$$

Vertical Asymptote(s):

$$x = -1$$

Hole(s):

$$(1, \frac{1}{2})$$

Horizontal Asymptote(s):

$$y = 0$$

x-intercept(s):

none

y-intercept(s):

$$(0, 1)$$

Range (use graph):

$$(-\infty, 0) \cup (0, \infty)$$

Directions: Sketch each rational function. Be sure to state all key features including domain/range and increasing/decreasing.

$$3) f(x) = \frac{x^2 - x - 6}{x^2 + x - 2}$$

$$\frac{(x-3)(x+2)}{(x+2)(x-1)} = \frac{x-3}{x-1}$$

$$\frac{x+3}{x-1} = 0$$

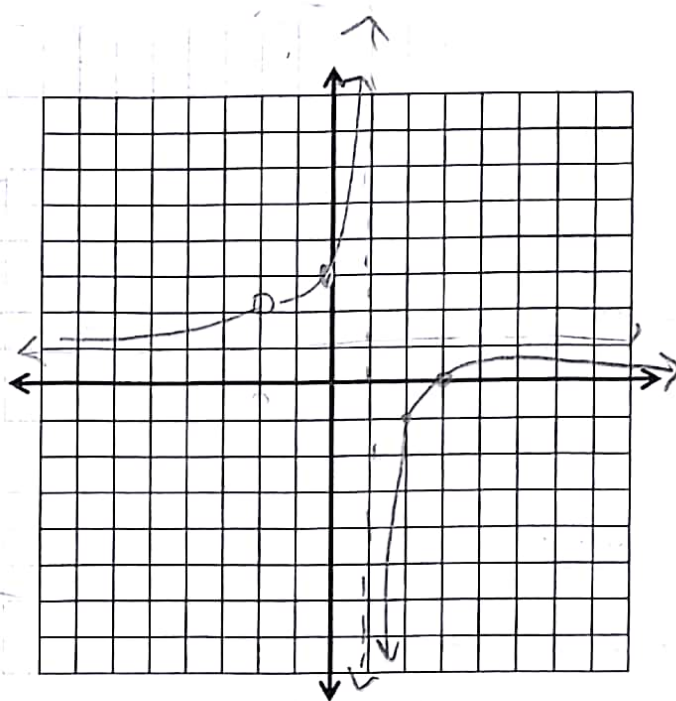
$$x+3=0$$

$$x=-3$$

x-int (-3, 0)

$$\frac{0+3}{0-1} = +3$$

y-int (0, 3)



VA $\rightarrow x-1=0$
 $x=1$

HA $\rightarrow \frac{x^2}{x^2} \Rightarrow y=1$

zeros $x+2=0$
 $x=-2$

$$\frac{-2+3}{-2-1} = \frac{-5}{-3} = \frac{5}{3} \quad (-2, \frac{5}{3})$$

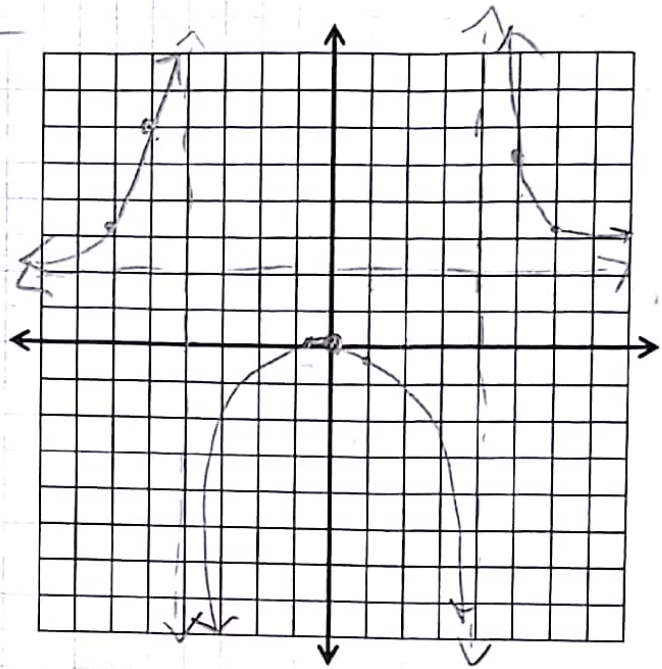
$$f(2) = \frac{2-3}{2-1} = \frac{-1}{1} = -1$$

$$4) f(x) = \frac{2x^2 - x}{x^2 - 16} = \frac{x(2x-1)}{(x-4)(x+4)}$$

$$\frac{x(2-x)}{(x-4)(x+4)} = 0$$

$$\frac{0(2-0)}{(0-4)(0+4)} = 0$$

$x(2x-1) = 0$
 $x = 0$ $2x-1 = 0$
 $+x = +\frac{1}{2}$
 $x = \frac{1}{2}$
 $(2, 0)$ $(\frac{1}{2}, 0)$
 $(4, 0)$



VA \rightarrow $x-4=0$ $x+4=0$
 $x=4$ $x=-4$

HA \rightarrow $\frac{2x^2}{x^2} \rightarrow y=2$

holes \rightarrow none

$$f(1) = \frac{1(2(1)-1)}{(1-4)(1+4)} = \frac{1(1)}{(-3)(5)} = -\frac{1}{15}$$

$$f(5) = \frac{5(2(5)-1)}{(5-4)(5+4)} = \frac{45}{9} = 5$$

$$f(6) = \frac{6(2(6)-1)}{(6-4)(6+4)} = \frac{66}{2(10)} = \frac{66}{20} = \frac{33}{10}$$

$$f(-5) = \frac{-5(2(-5)-1)}{(-5-4)(-5+4)} = \frac{+55}{(-9)(-1)} = \frac{+55}{9}$$

$$f(-7) = \frac{-7(2(-7)-1)}{(-7-4)(-7+4)} = \frac{105}{(-11)(-3)} = \frac{105}{33}$$

$$5) f(x) = \frac{x-2}{-2x^2+3x} = \frac{x-2}{x(-2x+3)}$$

$$\frac{x-2}{x(-2x+3)} = 0$$

$$x-2=0$$

$$x=2$$

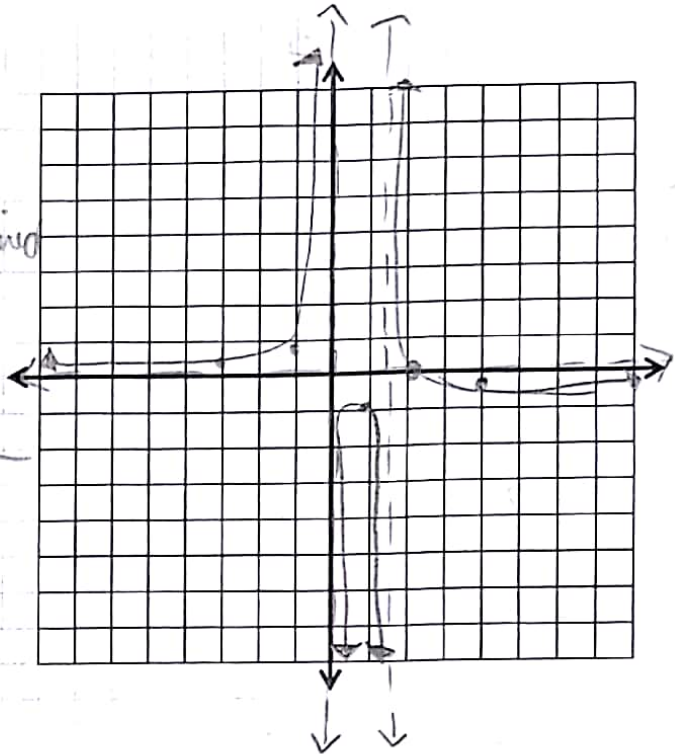
$$(2, 0)$$

x-int'

$$\frac{0-2}{0(-2(0)+3)} =$$

$$\frac{-2}{0} = \text{undefined}$$

y-int'



$$VA \rightarrow x(-2x+3) = 0$$

$$x=0 \quad -2x+3=0$$

$$\frac{-2x}{-2} = \frac{-3}{-2}$$

$$x = 3/2$$

$$HA \rightarrow \frac{x}{-2x^2} \rightarrow y=0$$

holes \rightarrow none

$$f(4) = \frac{4-2}{4(-2(4)+3)} = \frac{2}{4(-5)} = \frac{2}{-20} = -\frac{1}{10}$$

$$f(1) = \frac{1-2}{1((-2)(1)+3)} = -1$$

$$f(5) = \frac{5-2}{5(-2(5)+3)} = -1.5$$

$$f(-1) = \frac{-1-2}{-1(-2(-1)+3)} = .6$$

$$f(-3) = \frac{-3-2}{-3(-2(-3)+3)} = \frac{-5}{-3(9)} = \frac{-5}{-27} = \frac{5}{27}$$

$$6) f(x) = \frac{3}{x-2}$$

x-int

$$\frac{3}{x-2} = 0$$

$$3 \neq 0$$

none

y-int

$$\frac{3}{0-2} = -\frac{3}{2}$$

$$(0, -3/2)$$

$$VA \rightarrow x-2=0$$

$$x=2$$

$$HA \rightarrow \frac{3}{x} \rightarrow y=0$$

holes \rightarrow none

$$f(1) = \frac{3}{1-2} = \frac{3}{-1} = -3$$

$$f(-2) = \frac{3}{-2-2} = \frac{3}{-4}$$

$$f(3) = \frac{3}{3-2} = \frac{3}{1} = 3$$

$$f(5) = \frac{3}{5-2} = \frac{3}{3} = 1$$

