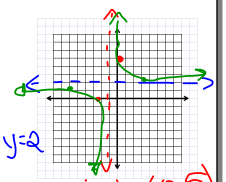
Slant Asymptotes

Try This: Sketch the following rational function. Be sure to find all key features!!

1) $f(x) = \frac{2x}{1+x}$



VA → $1+x=0$, $x=-1$

HA → $\frac{2x}{x} \to y=\frac{2}{1} \to y=2$

x-int → $(-\frac{5}{2},0)$

$\frac{5+2x}{1+x}=0$

$5+2x=0$

$2x=-5$

$x=-\frac{5}{2}$

y-int $(0,5)$

$\frac{5+2(0)}{(+0)}=\frac{5}{1}=5$

$f(-6)=\frac{-7}{5}=\frac{7}{5}$

$f(4)=\frac{13}{5}$

Slant Asymptotes:

-If the degree of the numerator is exactly one more than the degree of the denominator then the function has a **slant asymptote.**

-If there is a slant asymptote use **long division** to find the equation of the asymptote, the **dividend is the numerator** and the **divisor is the denominator,** the slant asymptote is the **quotient.**

without the remainder

Example: Find the slant asymptote.

1) $f(x) = \frac{x^2-x-2}{x-1}$

$x-1 \overline{)x^2-x-2}$

$-(x^2+x)$

-2

S.A. → $y=x$

2) $f(x) = \frac{x^2+x-1}{x+2}$

$x+2 \overline{)x^2+x-1}$

$x-1$

$-(x^2+2x)$

$-x-1$

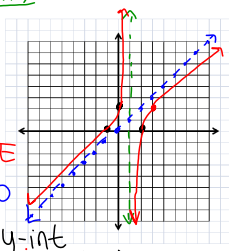
$-(-x-2)$

1

S.A. $y=x-1$

Now, let's graph them!

3) $f(x) = \frac{x^2-x-2}{x-1} = \frac{(x-2)(x+1)}{x-1}$



NO holes!

VA → $x-1=0$, $x=1$

HA → $\frac{x^2}{x}$ NONE

SA → $y=x+0$

x-int

$\frac{(x-2)(x+1)}{x-1}=\frac{0}{1}$

$(x-2)(x+1)=0$

$x-2=0$ $x+1=0$

$x=2$ $x=-1$

$(2,0)$ & $(-1,0)$

y-int

$\frac{(0-2)(0+1)}{0-1}=\frac{-2(1)}{-1}=2$

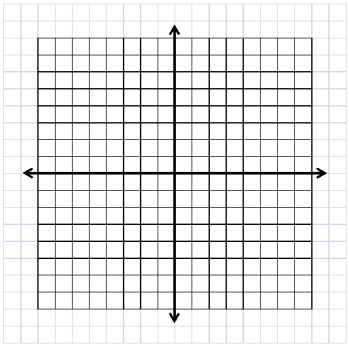
$(0,2)$

$f(3)=\frac{(3-2)(3+1)}{3-1}$

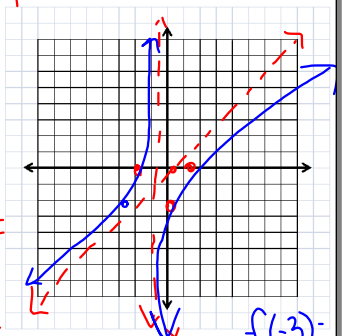
$\frac{(1)(4)}{2}=2$

*When you have a slant asymptote, there is no horizontal asymp

4) $f(x) = \frac{x^2-x-1}{x-3}$



5) $f(x) = \frac{x^2+x-2}{x+1} = \frac{(x-1)(x+2)}{x+1}$



no holes!

VA → $x+1=0$, $x=-1$

HA → $\frac{x^2}{x}$ NONE

SA → $y=x$

x-int $(-2,0)$, $(1,0)$

y-int $(0,-2)$

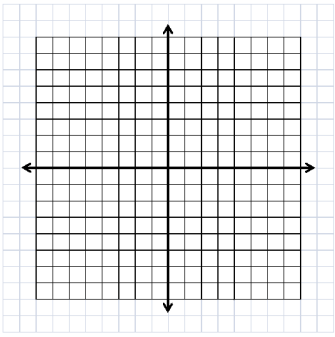
$x+1 \overline{)x^2+x-2}$

$-(x^2+x)$

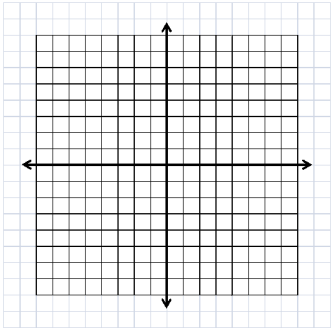
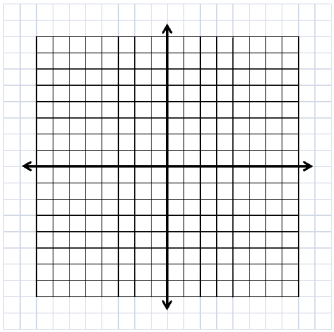
-2

$f(-3)=$

6) $f(x) = \frac{x^2+x-6}{x-3}$



7) $f(x) = \frac{x^2-2x-3}{x-2}$



8) $f(x) = \frac{x^2-2x-12}{x-5}$