Lesson 62 Objective: SWBAT use exponentials and logarithms to complete application problems.

Kickoff

Complete questions 9-10 and 23-24 on your homework sheet from last night!

9) $\log_3(x+4)+\log_3(x-2)=3$

$\log_3 (x+4)(x-2)=3$

$(x+4)(x-2)=3^3$

$x^2+2x-8=27$

$-27 \quad -27$

$x^2+2x-35=0$

$(x-5)(x+7)=0$

$x=5$ $\cancel{x=-7}$

10) $\log_4(x^2+3x)-\log_4(x+5)=1$

$\log_4 \frac{x^2+3x}{x+5}=1$

$\frac{4^1}{1}=\frac{x^2+3x}{x+5}$

$4x+20=x^2+3x$

$-4x-20 \quad -4x-20$

$0=x^2-x-20$

$0=(x-5)(x+4)$

$\boxed{x=5 \quad x=-4}$

23) $\log_5 4x^2+\log_5 2=\log_5 18$

$\log_5(4x^2)(2)=\log_5 18$

$\cancel{\log_5} 8x^2=\cancel{\log_5} 18$

$\frac{8x^2}{8}=\frac{18}{8}$

$\sqrt{x^2}=\sqrt{\frac{9}{4}}$

$x=\pm\frac{3}{2}$

24) $\log_5(x^2-1)-\log_5 4=\log_5 20$

$\cancel{\log_5}\frac{x^2-1}{4}=\cancel{\log_5}20$

$\frac{x^2-1}{4}=\frac{20}{1}$

$x^2-1=80$

$-80 \quad -80$

$x^2-81=0$

$(x+9)(x-9)=0$

$\boxed{x=-9 \quad x=9}$

① $\log_3(4-x)=\log_3(x+8)$

$4-x=x+8$

$+x \quad +x$

$4=2x+8$

$-8 \quad -8$

$\frac{-4}{2}=\frac{2x}{2}$

$\boxed{-2=x}$

② $\log_2 x-\log(\sqrt{x}-1)=2$

$\log_2\frac{x}{\sqrt{x}-1}=2$

$2^2=\frac{x}{\sqrt{x}-1}$

$\frac{4}{1}=\frac{x}{\sqrt{x}-1}$

$x=4(\sqrt{x}-1)$

$x=4\sqrt{x}-4$

$+4 \quad +4$

$\frac{x+4}{4}=\frac{4\sqrt{x}}{4}$

$\left(\frac{x+4}{4}\right)^2=(\sqrt{x})^2$

$\frac{x^2+8x+16}{16}=\frac{x}{1}$

$16x=x^2+8x+16$

$-16x \quad -16x$

$0=x^2-8x+16$

$(x-4)(x-4)$

$\boxed{x=4}$ $x=4$

③ $\log_5(x-3)=\log_5(\sqrt{x+3})$

$(x-3)^2=(\sqrt{x+3})^2$

$x^2-6x+9=x+3$

$-x \quad -3 \quad -x \quad -3$

$x^2-7x+6=0$

$(x-1)(x-6)=0$

$x\neq 1$ $\boxed{x=6}$

④ $\log_{x+3}\frac{x^3+x-2}{x}=2$

$(x+3)^2=\frac{x^3+x-2}{x}$

$\frac{x^2+6x+9}{1}=\frac{x^3+x-2}{x}$

$x^3+x-2=x^3+6x^2+9x$

$-x^3 \quad -x+2 \quad -x^3 \quad -x+2$

$0=6x^2+8x+2$

$2(3x^2+4x+1)$

$2(3x+1)(x+1)$

$x=-1/3$ $x=-1$

⑤ $\log_8(x+1)=16$

$8^{16}=x+1$

$-1 \quad -1$

$8^{16}-1=x$

too big!

2.81×10^{14}

⑥ $\log_{x+1} 64=3$

$((x+1)^3)^{\frac{1}{3}}=(64)^{\frac{1}{3}}$

$x+1=4$

$-1 \quad -1$

$\boxed{x=3}$

Applications of Exponential/Log Equations:

Annual Compounding:

$A=P(1\pm r)^t$

P = initial amount (Principle)
A = ending amount
r = rate (as a decimal)
t = number of years

Example #1: Ms. Zanfini bought a 2015 Honda CRV for \$30,000. It is known that the car has a depreciation (the decline in cash value) rate of 20%. The value of the car at any time can be determined by the formula $V=C(1-r)^t$, where V is the value of the car after t years, C is the original cost, and r is the rate of depreciation.

a) After 3 years how much is the car worth, *to the nearest cent*? After 10 years?

$A=?$ V

$V=C(1-r)^t$

$30,000(1-.20)^3$

$30,000(1-.2)^{10}$

b) How old is the car to the *nearest tenth of a year*, if the car has a value of \$15,000?

$t=?$

$15,000=30,000(1-.20)^t$

c) How old is the car to the *nearest tenth of a year*, if the car has a value of \$5,000?

Compounding continuously:

$$\underline{A} = Pe^{rt}$$

P = initial amount
A = ending amount
r = rate
t = time

Example: Susie invests $2000 in an account that is compounded continuously at an annual interest rate of 1.5%, according to the formula $A = Pe^{rt}$, where *A* is the amount accrued, *P* is the principal, *r* is the rate of interest, and *t* is the time, in years.

a) How much money will Susie have in her account after 3 years if she makes no additional deposits or withdrawals? After 18 years?

$A = ?$ $2{,}000\,e^{.015(3)}$ $2{,}000\,e^{.015(18)}$

b) Approximately how many years will it take for Susie's money to double?

$A = 2 \times 2{,}000$
$4{,}000$

$\frac{4{,}000}{2{,}000} = \frac{2{,}000\,e^{.015t}}{2{,}000}$

$\ln 2 = \ln e^{.015t}$

$\frac{\ln 2}{.015} = \frac{.015t \ln e}{.015}$

$46.2 \approx 47$ years

c) Approximately how many years will it take for Susie's money to triple?

Compounded Interest "n" times per year

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

P = initial amount
A = ending amount
r = rate
t = number of years
n = number of times it compounds per year

Example: Nick invests $1000 at Chase bank offering a 2% interest rate. Find the amount of the investment, to the nearest cent, at the end of 5 years if compounded…

Annually $n=1$	Semiannually $n=2$
Quarterly $n=4$	Monthly $n=12$
Daily $n=365$	

Population Growth/Decay

$$A = I(1 \pm r)^t$$

A= ending amount
I = initial population
r = rate
t = time in years

0 ← initial

Example: A town had a population of 27, 000 in the year $\boxed{2000}$. It is increasing at a rate of 2.5% each year. Find the year the population will be 50,000. ~~(hint: use the calculator to graph)~~

$A = I(1+r)^t$

$50{,}000 = 27{,}000(1+.025)^t$

Radioactive Decay

$$f(t) = S\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

F(t) = the amount of the substance that remains
S = Initial Mass
h = half life
t = time in years

Example: The half life of a radioactive element is 25 years. The initial mass is 10 grams.

A) Write an equation of the exponential function that models the radioactive decay of the element.

B) How many grams of the substance remains after 40 years?

C) How long will it take to have less than 1 gram left? (hint: use calculator to graph)

Homework/Classwork

Evens