

Lesson 76 Objective: SWBAT solve trig equations without a calculator.

Kickoff

Complete questions 6 and 7 on the worksheet from the desk!

6)  $1 = \cot^2 x$  F(0, 360)

Let  $x = \cot x$   $\tan x = 1$   $\tan x = -1$   
neq 45

$\sqrt{1} = \sqrt{x^2}$

$\pm 1 = x$

$\pm 1 = \cot x$

$\pm 1 = \tan x$

$\theta = 45$     $\theta = 135$   
 $\theta = 225$     $\theta = 315$

7)  $\tan^2 x + 3 \tan x + 1 = 0$

Let  $x = \tan x$

$x^2 + 3x + 1 = 0$

$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$X = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$

$\tan x = \frac{-3 + \sqrt{5}}{2}$     $\tan x = \frac{-3 - \sqrt{5}}{2}$

neq 21   neq 69

II  $\theta = 159$    II  $\theta = 111$   
 IV  $\theta = 339$    IV  $\theta = 291$

Recall the Exact Value Chart!

$\theta$	30 $\pi/6$	45 $\pi/4$	60 $\pi/3$
Sin $\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos $\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
Tan $\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Recall the Unit Circle!

$\theta$	0°, 360° 0, 2 $\pi$	90° $\frac{\pi}{2}$	180° $\pi$	270° $\frac{3\pi}{2}$
Sin $\theta$	0	1	0	-1
Cos $\theta$	1	0	-1	0
Tan $\theta$	0	DNE	0	DNE

Solving Trig Equations:

- 1) Substitute x for the given trig function.
- 2) Solve for x (if 2<sup>nd</sup> degree factor/quadratic equation)
- 3) Substitute the trig function back in for x  $\rightarrow$  check
- 4) Find the reference angle(s) and use it to find all solutions

Examples: Find all values of  $\theta$ , to the nearest degree, on the given interval.  $0 \leq \theta < 360$

1)  $\tan \theta = -1$   
 neq 45  
 II  $\theta = 135$   
 IV  $\theta = 315$

2)  $\cos \theta = \frac{\sqrt{2}}{2}$   
 II  $\theta = 45$   
 I  $\theta = 315$

3)  $2 \sin \theta - 1 = 0$   
 $\sin \theta = \frac{1}{2}$   
 neq 30  
 II  $\theta = 150$

4)  $(\sin \theta)(\cos \theta - 1) = 0$   $0 \leq \theta < 360$

Quadrantal

$\sin \theta = 0$     $\cos \theta - 1 = 0$

$\theta = 0$     $\theta = 0$   
 $\theta = 180$     $\theta = 360$

5)  $(2 \cos \theta + \sqrt{3})(\sin \theta + 1) = 0$

$2 \cos \theta + \sqrt{3} = 0$     $\sin \theta + 1 = 0$

$\cos \theta = -\frac{\sqrt{3}}{2}$     $\sin \theta = -1$

neq 30   neq 270

II  $\theta = 150$     $\theta = 270$   
 III  $\theta = 210$

Interval -  $0 < \theta < 2\pi$

6)  $\cos^2 \theta = \cos \theta$       7)  $2 \cos \theta \sin \theta = \sqrt{3} \sin \theta$

$\cdot \sqrt{3} \sin \theta - \sqrt{3} \sin \theta$   
 $2 \cos \theta \sin \theta - \sqrt{3} \sin \theta = 0$

$\sin \theta (2 \cos \theta - \sqrt{3}) = 0$

$\sin \theta = 0$

~~$\theta = 0$~~

~~$\theta = 2\pi$~~

$\theta = \pi$

$2 \cos \theta - \sqrt{3} = 0$

$\cos \theta = \frac{\sqrt{3}}{2}$

rel  $\frac{\pi}{6}, \frac{11\pi}{6}$

$\theta = \frac{\pi}{6}$

$\theta = \frac{11\pi}{6}$

$\frac{5\pi}{6} \quad \frac{7\pi}{6}$

$\frac{11\pi}{6} \quad \frac{13\pi}{6}$

8)  $4 \cos^2 \theta - 3 = 0$       9)  $2 \cos^2 \theta = 1 - \cos \theta$

$4x^2 - 3 = 0$

$\sqrt{x^2} = \frac{\sqrt{3}}{\sqrt{4}}$

$x = \pm \frac{\sqrt{3}}{2}$       I, IV

$\cos \theta = \pm \frac{\sqrt{3}}{2}$        $\leftarrow$  II, III

rel  $\frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$