Lesson 98 Objective: SWBAT understand and determine limits graphically.

Kickoff- Factor the following

$1 + 64x^3$

$\sqrt[3]{1} = 1 \quad \sqrt[3]{64x^3} = 4x$ SOAP

$(1 + 4x)((1)^2 - 1(4x) + (4x)^2)$

$(1 + 4x)(1 - 4x + 16x^2)$

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A **Limit** is a y value that a graph approaches, but may or may not reach.

Notation: The limit as x approaches c of f(x)

$\lim_{x \to c} f(x)$

* A ***limit exists*** *at x = c if and only if, the limit from the right = limit from the left* *

$\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$

A ***limit does not exist*** at x = c , ***If the limit from the right*** $\neq$ ***limit from the left.***

DNE

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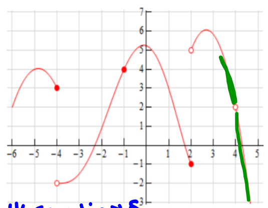
Example #1: Evaluate each limit based off of the graph f(x) below.



*continuous

a) $\lim_{x \to -4^-} f(x) = 3$

b) $\lim_{x \to -4^+} f(x) = -2$

c) $\lim_{x \to -4} f(x) =$ DNE

d) $f(-4) = 3$

e) $\lim_{x \to -1^-} f(x) = 4$

f) $\lim_{x \to -1^+} f(x) = 4$

g) $\lim_{x \to -1} f(x) = 4$

h) $f(-1) = 4$

i) $\lim_{x \to 2^-} f(x) = -1$

j) $\lim_{x \to 2^+} f(x) = 5$

k) $\lim_{x \to 2} f(x) =$ DNE

l) $f(2) = -1$

m) $\lim_{x \to 4^-} f(x) = 2$

n) $\lim_{x \to 4^+} f(x) = 2$

o) $\lim_{x \to 4} f(x) = 2$

p) $f(4) =$ DNE

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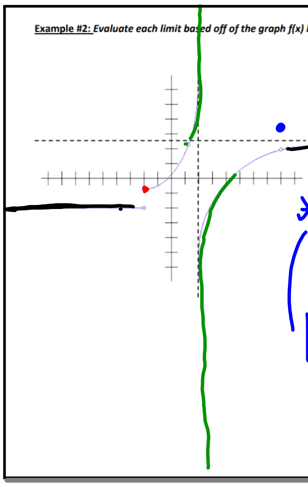
Example #2: Evaluate each limit based off of the graph f(x) below.



a) $\lim_{x \to -2^-} f(x) = -1$

b) $\lim_{x \to -2^+} f(x) = -2$

c) $\lim_{x \to -2} f(x) =$ DNE

d) $f(-2) = -1$

* e) $\lim_{x \to 8} f(x) = 2$

f) $f(8) = 4$

g) $\lim_{x \to 2^-} f(x) = -\infty$

h) $\lim_{x \to 2^+} f(x) = \infty$

i) $\lim_{x \to 2} f(x) =$ DNE

j) $f(2) =$ DNE

k) $\lim_{x \to -\infty} f(x) = -2$

l) $\lim_{x \to \infty} f(x) = 2.5$

$\lim_{x \to 8^-} = \lim_{x \to 8^+}$

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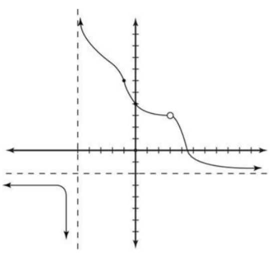
You Try: Evaluate each limit based off of the graph f(x) below.



a) $\lim_{x \to 3^+} f(x) =$

b) $\lim_{x \to 3^-} f(x) =$

c) $\lim_{x \to 3} f(x) =$

d) $f(3) =$

e) $\lim_{x \to -1} f(x) =$

f) $f(-1) =$

g) $f(0) =$

h) $\lim_{x \to -5^+} f(x) =$

i) $\lim_{x \to -5^-} f(x) =$

j) $\lim_{x \to -5} f(x) =$

k) $f(5) =$

l) $\lim_{x \to -\infty} f(x) =$

m) $\lim_{x \to \infty} f(x) =$

n) $\lim_{x \to 0} f(x) =$

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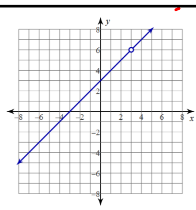
Limits Graphically

Given $f(x) = \frac{x^2 - 9}{x - 3}$, find the following limit graphically.



$\lim_{x \to 3} f(x) = 6$

Check the limit from the left and right

$\lim_{x \to 3^-} f(x) = 6$

$\lim_{x \to 3^+} f(x) = 6$

Limits Using Tables

Given $f(x) = \frac{x^2 - 9}{x - 3}$ find the $\lim_{x \to 3} f(x)$ using the table below

2 3 4

x	2.99	2.999	2.9999	3	3.0001	3.001	3.01
f(x)	5.99	5.999	5.9999	? 6	6.0001	6.001	6.01

Limit from the left — Limit from the right

$f(2.99) = \frac{(2.99)^2 - 9}{2.99 - 3}$

$f(3.01) = \frac{(3.01)^2 - 9}{3.01 - 3}$

$f(2.999) = \frac{(2.999)^2 - 9}{(2.999) - 3} = 5.999$

$f(3.001) = \frac{(3.001)^2 - 9}{(3.001) - 3} = 6.001$

$f(2.9999) = \frac{(2.9999)^2 - 9}{(2.9999) - 3} = 5.9999$

$f(3.0001) = \frac{(3.0001)^2 - 9}{(3.0001) - 3} = 6.0001$

Therefore, $\lim_{x \to 3} f(x) = 6$

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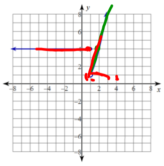
Limits Graphically

Find the following limits graphically for $f(x)=\begin{cases}4, & x\le 1\\ x^2, & x>1\end{cases}$



a. $\lim_{x\to 2} f(x) =$ 4

b. $\lim_{x\to 1} f(x) =$ DNE

Limits Using Tables

Using tables, find the following limits given that $\boxed{f(x)=\begin{cases}4, & x\le 1\\ x^2, & x>1\end{cases}}$

a. $\lim_{x\to 2} f(x)$

	Limit from the left →				← Limit from the right		
x	**1.99**	**1.999**	**1.9999**	**2**	**2.0001**	**2.001**	**2.01**
$f(x)$	3.96	3.996	3.9996	? 4	4.0004	4.004	4.04

$f(1.99) = (1.99)^2$ $f(2.01) = (2.01)^2$

$f(1.999) = (1.999)^2 = 3.996$ $f(2.001) = (2.001)^2 = 4.004$

$f(1.9999) = (1.9999)^2 = 3.9996$ $f(2.0001) = (2.0001)^2 = 4.0004$

Therefore, $\lim_{x\to 2} f(x) =$ 4

b. $\lim_{x\to 1} f(x)$

	Limit from the left →				← Limit from the right		
x	**0.99**	**0.999**	**0.9999**	**1**	**1.0001**	**1.001**	**1.01**
$f(x)$	4	4	4	?	1.0002	1.002	1.02

$f(.99) =$ 4 $f(1.01) = (1.01)^2 = 1.02$

$f(0.999) = 4$ $f(1.001) = (1.001)^2 = 1.002$

$f(0.9999) = 4$ $f(1.0001) = (1.0001)^2 = 1.0002$

Therefore, $\lim_{x\to 1} f(x) =$ DNE

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$\boxed{1-3}$, 12, 13

HW: 5-6, 10, 14

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