Review Sheet Solutions

(1a) $f(x) = x^2(x-9)^2(x+4)(x-2)$

$x^2=0$ $(x-9)^2=0$ $(x+4)=0$ $(x-2)=0$

$x=0$ $x-9=0$ $x=-4$ $x=2$

Bounce $x=9$ cross cross

Bounce

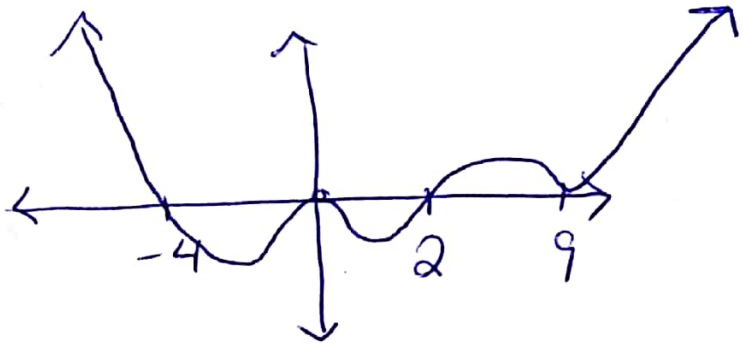
end behavior

$x^2(x)^2(x)(x) \to x^6$

D: 6 LC: 1

$x \to \infty$ $f(x) \to \infty$

$x \to -\infty$ $f(x) \to \infty$



(1b) $f(x) = (-x^3+x)(x^2+5x-6)$

$-x(x^2-1)(x+3)(x-2)$

$-x(x+1)(x-1)(x+3)(x-2)$

$-x=0$ $x+1=0$ $x+3=0$ $x-2=0$ $x-1=0$

$x=0$ $x=-1$ $x=+3$ $x=+2$ $x=1$

all cross!

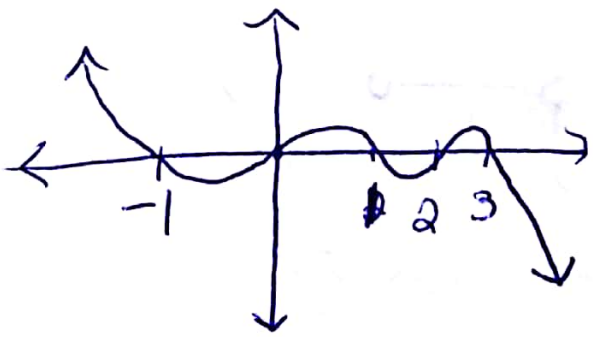
End Behavior

$x \to \infty$ $f(x) \to -\infty$

$x \to \infty$ $f(x) \to \infty$

$(-x^3)(x^2) = -x^5$

D: 5 LC: -1



② A polynomial can have at most n zeros and n-1 extremas.

③ $f(x) = x^3$

$g(x) = (x-2)^3 - 2$ right 2, down 2

④ a) LC: -4 D: 3

$x \to \infty \quad f(x) \to -\infty$

$x \to -\infty \quad f(x) \to \infty$

b) LC: 1 D: 4

$x \to \infty \quad f(x) \to \infty$

$x \to -\infty \quad f(x) \to \infty$

c) LC: 3 D: 5

$x \to \infty \quad f(x) \to \infty$

$x \to -\infty \quad f(x) \to -\infty$

d) LC: 5 D: 2

$x \to \infty \quad f(x) \to -\infty$

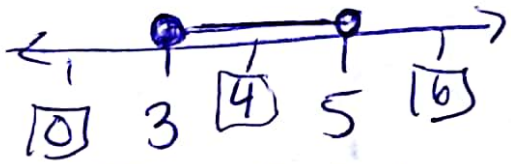
$x \to -\infty \quad f(x) \to -\infty$

⑤ a) $x^2 - 8x + 15 \leq 0$

$(x-3)(x-5) \leq 0$

$x-3=0 \quad x-5=0$

$x=3 \quad x=5$



Test Points

$0^2 - 8(0) + 15 \not\leq 0$

$15 \not\leq 0$

$4^2 - 8(4) + 15 \leq 0$

$-1 \leq 0$ ✓

$6^2 - 8(6) + 15 \leq 0$

$3 \not\leq 0$

b) $3x^2 - 14x - 5 > 0$

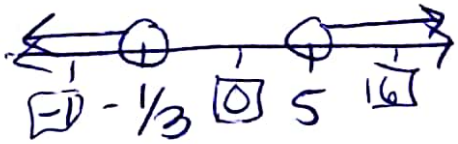
$(3x^2 - 15x) + (x - 5) > 0$

$3x(x-5) + 1(x-5)$

$(3x+1)(x-5) > 0$

$3x+1=0 \quad x-5=0$

$x = -1/3 \quad x=5$



Test Points

$3(-1)^2 - 14(-1) - 5 > 0$

$13 > 0$ ✓

$3(0)^2 - 14(0) - 5 > 0$

$-5 \not> 0$

$3(6)^2 + 4(6) - 5 > 0$

$19 > 0$ ✓

(6) a)
$$x-4 \overline{)\,5x^2-17x-12}\quad 5x+3$$
$-(5x^2+20x)$
$3x-12$
$-(3x-12)$
0

$5x+3$

b)
$$x-2 \overline{)\,x^4+5x^3+6x^2-x-2}\quad x^3+7x^2+20x+39$$
$-(x^4-2x^3)$
$7x^3+6x^2$
$-(7x^3-14x^2)$
$20x^2-x$
$-(20x^2-40x)$
$39x-2$
$39x-78$
76

$x^3+7x^2+20x+39+\frac{76}{x-2}$

(7) a) $f(x) = -4x^2+24x-41$

$\frac{f(x)+41}{-4} = \frac{-4x^2+24x}{-4}$

$\frac{f(x)}{-4} + \frac{41}{-4} \overset{+9}{=} x^2-6x+9$

$\frac{f(x)}{-4} - \frac{5}{4} = (x-3)^2 + \frac{5}{4}$
$+5/4$

$-4\left(\frac{f(x)}{-4}\right) = \left((x-3)^2+\frac{5}{4}\right)\cdot -4$

$f(x) = -4(x-3)^2-5$

Vertex $= (3,-5)$

axis of Symmetry $x=3$

b) $g(x) = \frac{1}{2}x^2+8x-1$

$(g(x)+1 = \frac{1}{2}x^2+8x)\ 2$

$2g(x)+2 = x^2+16x+64$
$+64$

$2g(x)+66 = (x+8)^2-66$
-66

$\frac{2g(x)}{2} = \frac{(x+8)^2-66}{2}$

$g(x) = \frac{1}{2}(x+8)^2-33$

Vertex $(-8,-33)$

axis of Symmetry $x=-8$

(8) a) $(3x^2-x+3)+(x^2+5x-7)$

$4x^2+4x-4$

b) $(x^2+5x-7)-(3x^2-x+3)$

$x^2+5x-7-3x^2+x-3$

$2x^2+6x-10$

c) $(3x^2-x+3)(2x-1)$

$6x^3-3x^2-2x^2+x+6x-3$

$6x^3-5x^2+7x-3$

(10) a) $g(x) = 3x^4 + 4x^3 - 3$

Put in Calculator (y=) and use table!

x	g(x)
-2	13
-1	-4
0	-3
1	4
2	77

The intervals of length 1 that must have zeros are (-2, -1) and (0, 1).

b) $f(x) = x^4 - 10x^2 + 2$

x	f(x)
-4	98
-3	-7
-2	-22
-1	-7
0	2
1	-7
2	-22
3	-7
4	98

The intervals of length 1 that must have zeros are (-4, -3), (-1, 0), (0, 1) and (3, 4).

(9) a)

$$\begin{array}{r|cccc} 1 & 2 & 6 & -14 & 9 \\ & \downarrow & 2 & 8 & -6 \\ \hline & 2 & 8 & -6 & \mid 3 \end{array}$$

$2x^2 + 8x - 6 + \frac{3}{x+1}$

b)

$$\begin{array}{r|cccc} -2 & 5 & 0 & 6 & 8 \\ & \downarrow & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & \mid -44 \end{array}$$

$5x^2 - 10x + 26 + \frac{44}{x+2}$

(11) a) An even multiplicity will cause the graph of a polynomial to be tangent (bounce) at that value of zero on the x-axis.

b) An odd multiplicity will cause the graph of a polynomial to cross the x-axis at that value.

Name:_____ Date:_____

Pre-Calculus Review #3

1. For each polynomial function find the zeroes, sketch a graph of the function and describe the end behavior. NO CALCULATOR

   a. $f(x)=x^2(x-9)^2(x+4)(x-2)$ b. $f(x)=(-x^3+x)(x^2+5x-6)$

2. What are the most zeros and extremas a polynomial function of degree n can have?

3. If $f(x)=x^3$
   $g(x)=(x-2)^3-2$
   state the transformation from f to g and graph both functions.

4. Use the Leading Coefficient Test to describe the left and right hand behaviors of the following functions:

   a. $f(x)=-4x^3+2x^2-5$
   b. $f(x)=x^4+5x^3+3x^2-4x+1$
   c. $f(x)=3x^5+9x^4-3$
   d. $f(x)=-5x^2+3x-1$

5. Solve and graph the solution on a number line:
   a) $x^2-8x+15\le 0$
   b) $3x^2-14x-5>0$

6. Use long division to find the quotient:
   a) $5x^2-17x-12$ *by* $x-4$
   b) $x^4+5x^3+6x^2-x-2$ *by* $x-2$

7. Write the quadric function in vertex form and then identify the vertex and axis of symmetry.
   a) $f(x)=-4x^2+24x-41$

   b) $g(x)=\frac{1}{2}x^2+8x-1$