

Final Review #5

1)

$$g(x) = \begin{cases} -x^2 + 4 & x \leq 1 \\ 6x - 1 & x > 1 \end{cases}$$

① $f(1) = -1^2 + 4 = 3$

② $\lim_{x \rightarrow 1^-} -x^2 + 4 = 3$ $\lim_{x \rightarrow 1} = \text{DNE}$
 $\lim_{x \rightarrow 1^+} 6x - 1 = 5$

NOT CONTINUOUS!

2)

$$h(x) = \begin{cases} 5x - 7 & x \leq -1 \\ b + 4x & x > -1 \end{cases}$$

① $f(-1) = 5(-1) - 7 = -12$

② $\lim_{x \rightarrow -1^-} 5x - 7 = -12$
 $\lim_{x \rightarrow -1^+} b + 4x = b - 4$

$$\lim_{x \rightarrow -1^-} h(x) = \lim_{x \rightarrow -1^+} h(x)$$

$$-12 = b - 4$$

$$\begin{matrix} +4 & +4 \\ \hline 16 = b \end{matrix}$$

3)

a) $\lim_{h \rightarrow 0} \frac{2(x+h)^2 - 3(x+h) - [2x^2 - 3x]}{h}$

$$\lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$$

$$\lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 3h}{h}$$

$$\lim_{h \rightarrow 0} 4x + 2h - 3 = \boxed{4x - 3 = f'(x)}$$

b) $f(-3) = 2(-3)^2 - 3(-3) = 27$
 $f'(-3) = 4(-3) - 3 = -15$

T: $y - 27 = -15(x + 3)$

N: $y - 27 = \frac{1}{15}(x + 3)$

4)

$$x^2 + 9y^2 + 8x - 18y + 16 = 0$$

$$x^2 + 8x + 9y^2 - 18y = -16$$

$$x^2 + 8x + 9(y^2 - 2y) = -16$$

$$\frac{(x+4)^2 + 9(y-1)^2}{9} = \frac{-16 + 16 + 9}{9}$$

$$\frac{(x+4)^2}{9} + \frac{(y-1)^2}{1} = 1$$

Ellipse
 horizontal
 Center $(-4, 1)$
 $a = 3$
 $b = 1$

vertices
 $(-7, 1)$ $(-4, 0)$
 $(-1, 1)$ $(-4, 2)$

Foci $a^2 = b^2 + c^2$
 $9 = 1 + c^2$
 $\sqrt{8} = c$
 $2\sqrt{2} = c$

$(-4 \pm 2\sqrt{2}, 1)$

5)

$$\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$$

$$\frac{\frac{1}{\sin x}}{\frac{1}{\sin x}} - \frac{\frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x}} = 1 \quad \text{Pythagorean Identity}$$

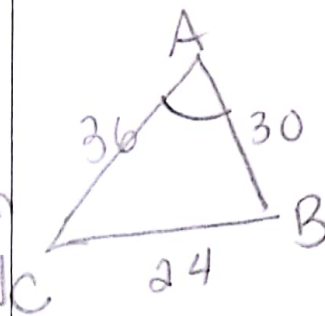
$$\frac{1}{\sin x} \cdot \frac{1}{\sin x} - \frac{\cos x \cdot \cos x}{\sin x \cdot \sin x} = 1$$

$$\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} = 1$$

$$\csc^2 x - \cot^2 x = 1$$

$$1 = 1 \checkmark$$

6)



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$24^2 = 36^2 + 30^2 - 2(36)(30) \cos A$$

$$576 = 2196 - 2160 \cos A$$

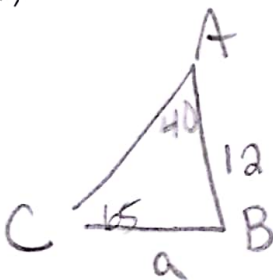
$$2196 - 2196$$

$$-1680 = -2160 \cos A$$

$$\frac{-1680}{-2160} = \frac{-2160 \cos A}{-2160}$$

$$\frac{3}{4} = \cos A \quad \boxed{\angle A = 41^\circ}$$

7)



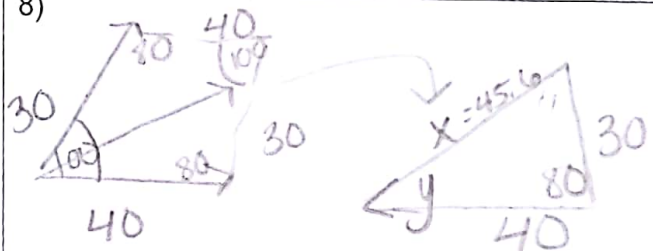
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 40} = \frac{12}{\sin 65}$$

$$\frac{12 \sin 40}{\sin 65} = \frac{a \sin 65}{\sin 65}$$

$$\boxed{9 = a}$$

8)



a) "x"

$$x^2 = 30^2 + 40^2 - 2(30)(40) \cos 80$$

$$\sqrt{x^2} = \sqrt{2083.244374}$$

$$\boxed{x = 45.6 \text{ lbs}}$$

b) "y"

$$\frac{45.6}{\sin 80} = \frac{30}{\sin y}$$

$$\frac{30 \sin 80}{45.6} = \frac{45.6 \sin y}{45.6}$$

$$.6478998375 = \sin y$$

$$\sin^{-1}(.647...) = \boxed{40^\circ}$$

9)

$$4x - 3y + z = -10$$

$$2x + y + 3z = 0$$

$$-x + 2y - 5z = 17$$

$$\textcircled{1} \quad 4x - 3y + z = -10$$

$$\textcircled{2} \quad -2(2x + y + 3z = 0)$$

$$\downarrow$$

$$\begin{array}{r} 4x - 3y + z = -10 \\ -4x - 2y - 6z = 0 \\ \hline A = -5y - 5z = -10 \end{array}$$

$$\textcircled{2} \quad 2x + y + 3z = 0$$

$$\textcircled{3} \quad 2(-x + 2y - 5z = 17)$$

$$\downarrow$$

$$\begin{array}{r} 2x + y + 3z = 0 \\ -2x + 4y - 10z = 34 \\ \hline B = 5y - 7z = 34 \end{array}$$

$$-5y - 5z = -10$$

$$5y - 7z = 34$$

$$-12z = 24$$

$$z = -2$$

$$B = 5y - 7(-2) = 34$$

$$5y + 14 = 34$$

$$5y = 20$$

$$y = 4$$

$$\textcircled{3} \quad -x + 2(4) - 5(-2) = 17$$

$$-x + 8 + 10 = 17$$

$$-x + 18 = 17$$

$$-x = -1$$

$$x = 1$$

$$\boxed{(-1, 4, -2)}$$